Moment Closure Description of Polydisperse, Polykinetic and Evaporating Liquid Sprays

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Abstract: The description of liquid sprays is commonly modelled using a statistical approach and can be described using a droplet number density function (NDF) which evolves according to the Williams-Boltzmann equation. Approximate solutions to the Williams-Boltzmann equation can be obtained either using Lagrangian particle methods or Eulerian moment closure methods. While Lagrangian methods are relatively robust and are applicable for a wide range of sprays in the disperse spray regime, the method can potentially be computationally expensive and require a large number of particles in order to obtain converged statistics. Conversely, moment closure methods operate within an Eulerian framework which can take advantage existing techniques to reduce the computational cost, such as load balancing and mesh adaptation strategies. In addition, an Eulerian framework is consistent with descriptions of dense spray regime and background carrier phase and the models can be coupled with relative ease. In this paper, an extended model based on the 10-moment anisotropic Gaussian (AG) closure from kinetic theory is presented. Additional size and size-velocity moments are introduced in order to describe the spray polydisperity and droplet size-velocity correlations. Closure of the model is achieved by assuming functional forms of the size-conditioned moments based on entropy maximization and polynomial expressions. Finally, several representative test problems are presented to illustrate the predictive capabilities of the model, including: an example of particle trajectory crossing (PTC), an evaporating jet using the d^2 evaporation law and a liquid jet in a crossflow for a general drag law.

Keywords: Multiphase Flows, Computational Fluid Dynamics, Moment Closures, Liquid Sprays.

1 Introduction

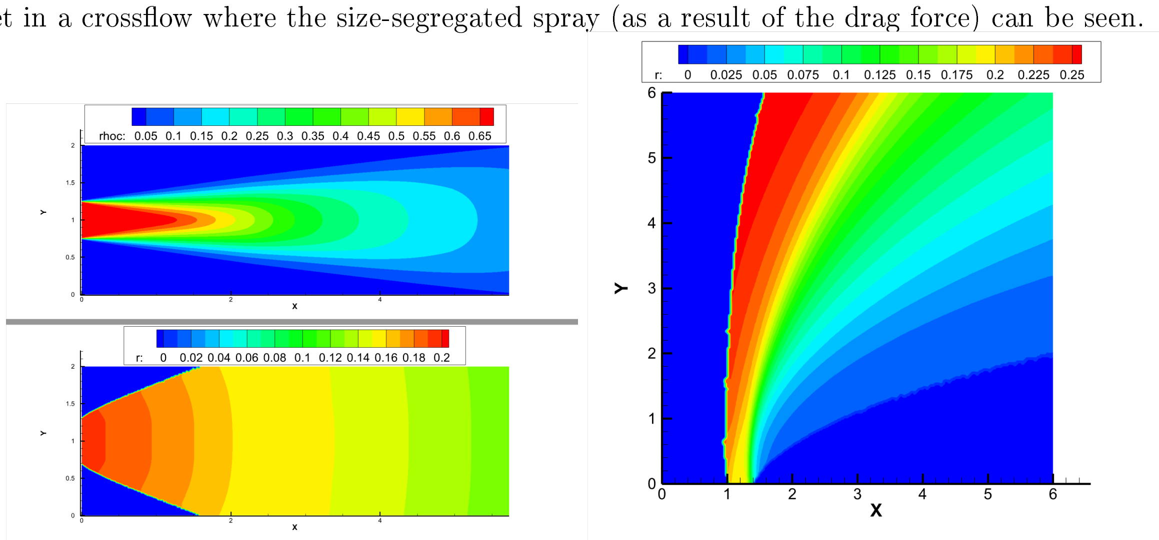
The application of liquid spray atomization to the fuel injection systems of gas turbine engines burning hydrocarbon fuels is of particular interest, where efforts are underway to lower emissions of carbon dioxide (CO_2), mono-nitrogen oxides (NO_X), as well as soot particulate matter. Finer atomization of fuel droplets can promote both better fuel-air mixing and hence cleaner combustion. Crucial to the development of improved fuel injection systems is the ability to accurately model the spray atomization process using computational fluid dynamic (CFD) simulations. Accurate, affordable, and reliable computational tools are required to explore different design configurations prior to the prototyping process.

In the proposed moment closure method, a polydisperse, polykinetic, Eulerian-based model for polydisperse and polykinetic liquid sprays with evaporation is developed herein, along with a robust finite-volume method for the numerical solution of the resulting system of hyperbolic moment equations. The model provides approximate solutions to the Williams-Boltzmann equation for disperse sprays and is based on the 10-moment Anisotropic Gaussian (AG) velocity moment closure [3, 1, 2, 4] which presents many desirable mathematical properties, such as strict hyperbolicity and moment realizability.

2 20-Moment Anisotropic Gaussian (AG) Moment Closure

The moment closure description of sprays can be derived from the Williams-Boltzmann equation [6]. In this study, the sprays are taken to be dilute and coalescence, breakup and collisions are neglected. The focus is placed on treatments of drag and evaporation arising from the interaction of the spray droplets with the background gas. General drag laws are used to prescribe the droplet drag and, in the current study, the evaporation processes are described using a d^2 evaporation law [5].

Preliminary results of the 20-moment closure description are depicted in Figure 1, where the initial size distributions are uniform for both cases. The effects of evaporation on the total number density and mean droplet radius can be seen in Figure 1(a), where the droplet number density and mean radius decreases as the evaporation takes place. Figure 1(b) shows a liquid jet in a crossflow where the size-segregated spray (as a result of the drag force) can be seen.



(a) Number Density (top) and Mean Radius (bottom).

(b) Mean Radius.

Figure 1: (a) Evaporating jet, (b) Liquid jet in a cross flow.

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