# Accelerating the Convergence of a Compressible RANS Solver for All Mach Numbers

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**Abstract:** Aerodynamic design often involves accounting for flow conditions that span low subsonic to supersonic Mach numbers. Compressible flow solvers encounter reduced accuracy and efficiency at low Mach numbers. To address these issues, we propose a characteristic time-stepping approach in the context of an approximate Newton–Krylov method with pseudo-transient continuation. This method will be a robust and efficient method for converging the Reynolds-averaged Navier–Stokes equations across all Mach numbers.

*Keywords:* Local Preconditioning, Pseudo-Transient Continuation.

## 1 Introduction

Aerodynamic design often requires evaluating performance at a wide range of flow conditions. For example, the flight envelope of a supersonic transport aircraft can span Mach numbers from 0.25 to 2.0. In addition, flow fields in certain applications can feature a range of Mach numbers at a single flow condition. Helicopter rotors and large wind turbines experience low subsonic flow at blade roots and potentially supersonic flow at the tips. Nacelles in crosswind can experience supersonic flow around the inlet lips despite nearly incompressible flow outside the nacelle. This motivates the need for an efficient solver for all Mach number flows. We propose a characteristic time-stepping method for an implicit Reynolds-averaged Navier–Stokes (RANS) solver.

## 2 Characteristic Time-Stepping

We develop the proposed method starting from an approximate Newton–Krylov (ANK) solver with pseudo-transient continuation [1]. Each iteration for the ANK solver can be written as

$$\left[\frac{I}{\Delta t^{(n)}} + \left(\frac{\partial R}{\partial U}\right)^{(n)}\right] \Delta U^{(n)} = -R(U^{(n)}), \tag{1}$$

where $R$ is the residual vector, $U$ is the state vector, and $\Delta t$ is the local time step. For characteristic time-stepping, we replace $\Delta t$ with a general time-stepping matrix $T$:

$$\left[\left(T^{-1}\right)^{(n)} + \left(\frac{\partial R}{\partial U}\right)^{(n)}\right] \Delta U^{(n)} = -R(U^{(n)}) \tag{2}$$

To construct the time-stepping matrix, we use the work by van Leer et al. [2], who introduced the characteristic time-stepping approach for the Euler equations. Characteristic time-stepping propagates each characteristic at a different speed, which reduces the stiffness in the system and improves the solver's convergence rate at both high and low Mach numbers. This approach can also be interpreted as local preconditioning. In terms of the Euler symmetrizing variables, $\tilde{U} = [p/(\rho c), u, v, w, s]^T$, and in the flow-aligned coordinate frame, the local preconditioning matrix is

$$P = \begin{bmatrix} \frac{\tau}{\beta^2}M^2 & -\frac{\tau}{\beta^2}M & 0 & 0 & 0 \\ -\frac{\tau}{\beta^2}M & \frac{\tau}{\beta^2}+1 & 0 & 0 & 0 \\ 0 & 0 & \tau & 0 & 0 \\ 0 & 0 & 0 & \tau & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where

$$\beta = \begin{cases} \sqrt{1-M^2}, M < 1 \\ \sqrt{M^2-1}, M \geq 1 \end{cases}, \qquad \tau = \begin{cases} \sqrt{1-M^2}, M < 1 \\ \sqrt{1-M^{-2}}, M \geq 1 \end{cases}. \tag{4}$$

The corresponding time-stepping matrix is given by

$$\tilde{T} = \text{CFL} \cdot \Delta V \cdot P. \tag{5}$$

Finally, we transform the matrix to be in terms of the conservative variables,

$$U = [\rho, \rho u, \rho v, \rho w, \rho E]^T, \tag{6}$$

and to Cartesian coordinates with transformation matrices $M$ and $Q$ (which we omit the full forms of for brevity):

$$T = MQ\tilde{T}Q^{-1}M^{-1}. \tag{7}$$

## 3 Proposed Work

In this work, we will implement the characteristic time-stepping approach described above in a Reynolds-averaged Navier–Stokes (RANS) solver. We will run RANS simulations for different configurations over a range of Mach numbers using the baseline local time-stepping method and characteristic time-stepping and compare the accuracy, efficiency, and robustness of the two methods. We expect that combining the local preconditioning provided by characteristic time-stepping with a pseudo-transient continuation method will result in a fast, general purpose RANS solver.

## References


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