A shock capturing sub-filter scale Legendre spectral viscosity (LSV) closure applied to high-order flux reconstruction schemes

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1 Introduction

In recent years, considerable effort has been put into the development of discontinuous high-order methods capable of solving a wide range of PDE's, including the Navier-Stokes equations. Such methods display a high convergence rate and, in general, require smaller grid resolutions to achieve accurate results when compared to the low-order counterpart. The latter, however, are more robust due to the inherent dissipation embedded in the low-order numerics.

The presence of high-gradients in the computational domain, such as shock-induced discontinuities, is particularly challenging for high-order schemes and dissipation must be introduced to avoid numerical instabilities. Nonetheless, this step must be performed meticulously so that the large scales present are minimally affected. A careless introduction of a dissipation term to stabilize a shock could deteriorate a large band of the spectrum of resolvable turbulence when both phenomena happen concomitantly, for example.

The current work devises a method for stabilizing nonlinear equations in the context of discontinuous high-order flux reconstruction (Huynh, 2007) methods by extending the Large Eddy Simulation (LES) framework, based on solving the filtered quantities and modeling of the sub-filter scale (SFS) energy flux, to capturing any discontinuity present in the flow. In that way, the introduced dissipation is close to the minimum necessary to maintain the filtered solution.

2 Problem Statement

A novelty is introduced in the Legendre spectral viscosity (LSV) closure model, where a projection of the discontinuous solution within each element onto the set of Legendre basis functions is used to inform the magnitude of the dissipation and to modulate its strength at different scales. This results in a SFS dissipation that is concentrated at the higher Legendre modes and that dynamically activates only in cells where nonlinear behavior is important. It is a further development built upon the Quasi-Spectral Viscosity (QSV) method (Sousa and Scalo, 2021) capable of modeling shocks and turbulence at the same time using high-order finite-difference operators. The full steps required for implementing the LSV method will be introduced in the final version of the manuscript.

The proposed method is tested in canonical shock-dominated flow setups in both one and two dimensions. Shown here are the results for 1D Burgers' problem (figure 1) and a 2D inviscid shock-vortex interaction (figure 2). These showcase robustness, being able to capture 1D shocks within a single cell with very high orders, and multidimensional capability.

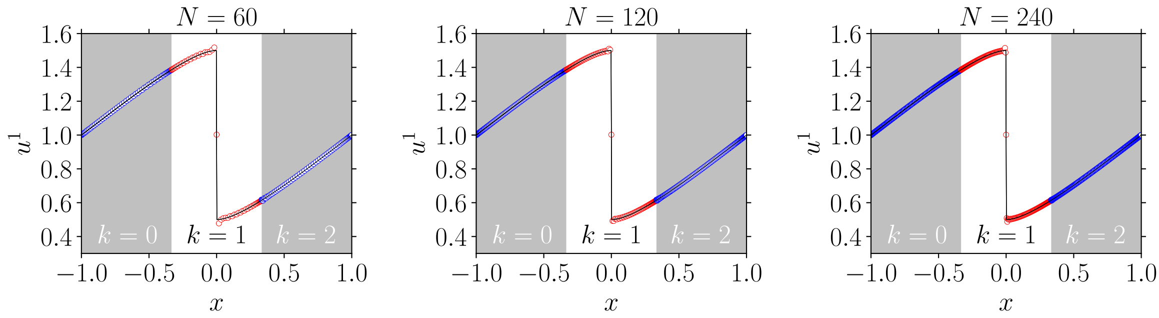

Figure 1: Flux reconstruction numerical solution of the Burger's equation using the LSV closure for very high-order polynomials. In this case $K = 3$ cells are used to discretize the domain and, in turn, each cell is solved with polynomial orders of $N = [60, 120, 240]$. The initial condition, $u^1(x, t = 0) = 1 + \frac{1}{2}\sin(\pi x)$, is solved up to $t = 1$ in a periodic domain $x^1 \in [-1, 1]$. The analytical solution is shown in fine solid black lines and the numerical results at the solution points are show in alternating red or blue circular markers. Regions corresponding to different cells are periodically shaded.

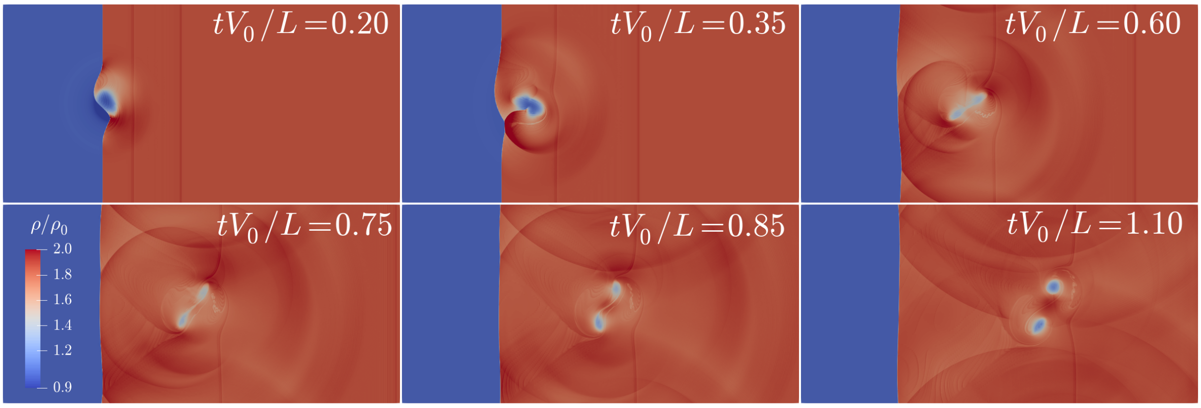

Figure 2: LSV simulation of an inviscid strong shock-strong vortex interaction performed with $N = 7$ polynomial order in both directions and $K_x \times K_y = [256 \times 128]$ cells. Six different time instants are shown to illustrate the system's development.

3 Conclusion and Future Work

Further test cases and an in depth analysis of the novel proposed model will be presented in the final manuscript.

References

H. Huynh. A Flux Reconstruction approach to high-order schemes including discontinuous Galerkin methods. *AIAA Paper*, 2007-4079:1–42, 2007. 18th AIAA Computational Fluid Dynamics Conference, Miami, FL, Jun. 25–28, 2007.

V. C. B. Sousa and C. Scalo. A unified Quasi-Spectral Viscosity (QSV) approach to shock capturing and large-eddy simulation, 2021.