Comparative Assessment of Accuracy of Shock-Capturing Schemes in Terms of Local-Truncation-Error

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Excessive numerical dissipation, which is considered an inherent limitation of the CFD method, dissipates the physical phenomena, weakens their intensity, and even makes them invisible. Especially in vortex-dominated compressible flowfields such as rotorcraft flowfield, accurate prediction of performance is difficult because numerous vortices effective to aerodynamic performance vanish due to the numerical dissipation. Shock-capturing scheme of low numerical dissipation can be an adequate approach for precise prediction. However, shock-capturing schemes do not promise the same performance in the discretized domain even if they have the same theoretical accuracy. This is because the non-linear part of each scheme locally deteriorates the accuracy according to grid quality and flow characteristics. Therefore, it is necessary to assess each scheme in terms of local accuracy and identify which characteristics are required for accurate prediction. In this study, a local-order-of-accuracy index (LAI) is newly defined to quantitatively measure the actual accuracy of spatial discretization scheme. The LAI consists of the product and sum of n^{th}-order error measures, which can be calculated through dot product of two vectors; 1) the truncation error coefficient of each stencil, and 2) the polynomial weight of each high-order scheme. Through several benchmark tests, the LAI analysis indicates two requisites for accurate and efficient prediction; 1) a sophisticated function of smoothness inherent in each scheme, and 2) hybrid central-upwind characteristics. Details of the LAI analysis results are presented including actual rotorcraft application problems.

Keywords: Shock-Capturing Schemes, Rotorcrafts, Local-order-of-accuracy, Truncation error, Aeroacoustics

1 Introduction

Shock capturing schemes apply artificial dissipation to deal with the shock robustly, but this has the disadvantage of poor accuracy in the incompressible area. The non-linear function inherent in each scheme is the main factor of degradation of local accuracy. The original role of this function is to robustly deal with discontinuity. The problem arises when this function work where discontinuity does not exist. If the grid quality is poor or the flow changes rapidly, the scheme can erroneously judge a discontinuity. This brings unnecessary numerical dissipation and deteriorates the local accuracy. For more efficient and accurate performance prediction, the shock-capturing scheme should have theoretical accuracy in as many domains as possible. Therefore, in this study, local truncation error analysis was conducted to analyze the local accuracy expressed in each flowfield. The essential requirements of shock-capturing scheme were examined for precise prediction of aerodynamic and aeroacoustic performance.

2 Local-order-of-accuracy Index (LAI) and Its Application

The cell interface quantity can be reconstructed as shown in Eq. 1 using r stencils to the left and $(n-r)$ stencils to the right based on the cell face, $\frac{1}{2}$. $\bar{q}_m$ is the averaged quantity of the m^{th} cell. $\boldsymbol{c}$ is the weights of each stencil, and $q_{\frac{1}{2},exact}$ means an exact physical quantity at the cell interface.

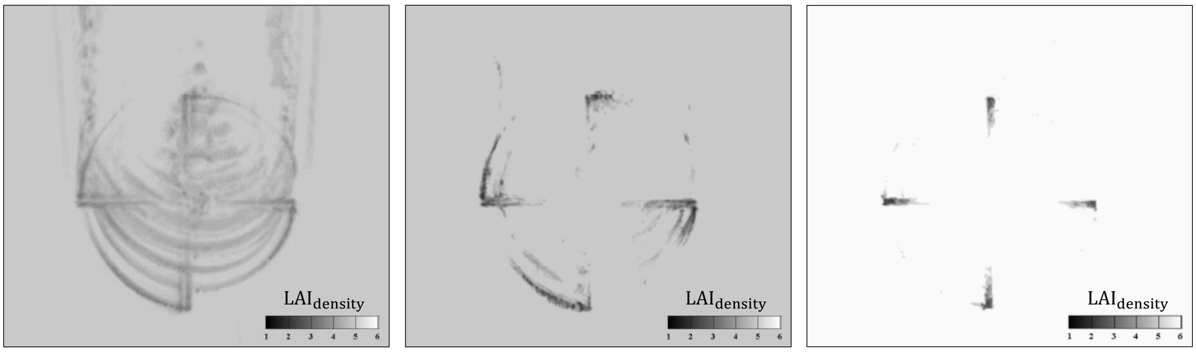
$$q_{\frac{1}{2}} = \sum_{m=-(r-1)}^{(n-r)} c_m \bar{q}_m = q_{\frac{1}{2},exact} + O(\Delta x^n) \tag{1}$$

Through Taylor series expansion analysis, truncation error ($\boldsymbol{e_i}$) of interface quantity can be obtained. Local-order-of-accuracy index (LAI) can be defined as Eq. 2.

$$\begin{aligned} \mathrm{LAI} &= 1 + \sum_{k=1}^{\mathrm{n}} \prod_{i=1}^{k} \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_i}|}{|\Gamma_i|}\right)^p \\ &= 1 + \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_1}|}{|\Gamma_1|}\right)^p + \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_1}|}{|\Gamma_1|}\right)^p \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_2}|}{|\Gamma_2|}\right)^p + \cdots + \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_1}|}{|\Gamma_1|}\right)^p \cdots \left(1 - \frac{|\boldsymbol{c}^{\boldsymbol{T}} \cdot \boldsymbol{e_n}|}{|\Gamma_n|}\right)^p \end{aligned} \tag{2}$$

Local truncation errors can be compared through LAI by calculating the weights($\boldsymbol{c}$) applied to the stencil according to each scheme. Details will be presented in the full paper.

The results of the HART II problem using the fifth-order accurate low-dissipation shock-capturing schemes are shown in Fig. 1. LAI contours are presented. The aeroacoustic results are shown in Fig. 2. It can be seen that even if the same order-of-accuracy scheme is used, completely different results are obtained in noise prediction. Details will be explained in the full paper.



a) LAI$_{\text{density}}$, WENO-JS **b) LAI$_{\text{density}}$, eMLP** **c) LAI$_{\text{density}}$, eMLP-VC**

Figure 1. Contours of local-order-of-accuracy index (HART II, 1.0chord above the rotor disk plane).

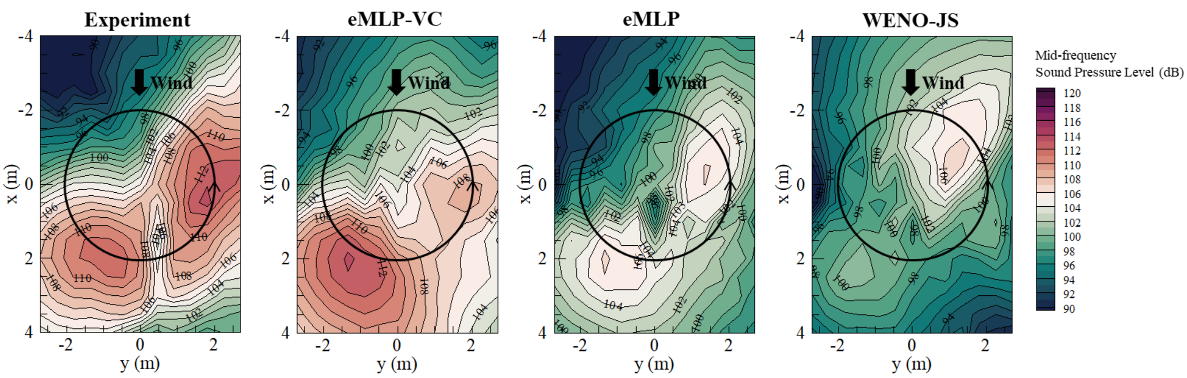


Figure 2. Noise map comparison (HART II, Mid-frequency SPL).

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