Laminar and Turbulent Behavior Captured by A 3-D Kinetic-Based Discrete Dynamic System

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Abstract: We derived the first-ever 3-D kinetic-based discrete dynamic system (DDS) from lattice Boltzmann equation (LBE) for incompressible flows through a Galerkin procedure. The DDS involves four bifurcation parameters including the relaxation time from the LBE and wavevector components from the Fourier space. Numerical simulations are presented in terms of time series and power spectra. The DDS can capture laminar behaviors of periodic, subharmonic, n-period, and quasi-periodic and turbulent behaviors of noisy periodic with harmonic, noisy subharmonic, noisy quasi-periodic, and broadband power spectra. We expect to apply this DDS in the large eddy simulation of turbulent pulsatile flows to provide dynamic sub-grid scale information.

Keywords: Lattice Boltzmann method, poor-man's lattice Boltzmann equation, time series, power spectra, turbulence modeling.

1 Introduction

Discrete dynamical systems (DDSs) have long been of interest for their ability to capture complicated turbulent-like behaviors while being very simple from a mathematical standpoint[1]. The "poor man's Navier--Stokes (PMNS) equation", introduced by Frisch[2], is an established DDS derived from the incompressible Navier--Stokes (N-S) equations. However, there are deficiencies in the DDS derived from N-S equations that are limited to small Knudsen numbers. In this work, we derive a first-ever 3-D kinetic-based DDS, i.e., "poor man's lattice Boltzmann (PMLB) equation" using the lattice Boltzmann method (LBM)[3]. We perform numerical experiments to explore the capability of the DDS to predict both laminar and turbulent flow behaviors.

2 Formulation of the 3-D kinetic-based DDS

The formulation of the PMLB equation consists of the following steps:

1) Decompose the distribution function f_i (i=0, 1,…,b) into large scale ($\tilde{f}_i$) and sub-grid scale (SGS) (f_i^*)
2) Perform a Fourier expansion for the distribution function to represent the large and small scales as

$$f_i(\vec{x},t) = \sum_{\vec{k}=0}^{\infty} a_{i,\vec{k}}(t)\varphi_{i,\vec{k}}(\vec{x}) = \underbrace{\sum_{\vec{k}=0}^{\vec{N}} a_{i,\vec{k}}(t)\varphi_{i,\vec{k}}(\vec{x})}_{\tilde{f}_i} + \underbrace{\sum_{\vec{k}=\vec{N}+\vec{I}}^{\infty} a_{i,\vec{k}}(t)\varphi_{i,\vec{k}}(\vec{x})}_{f_i^*}$$

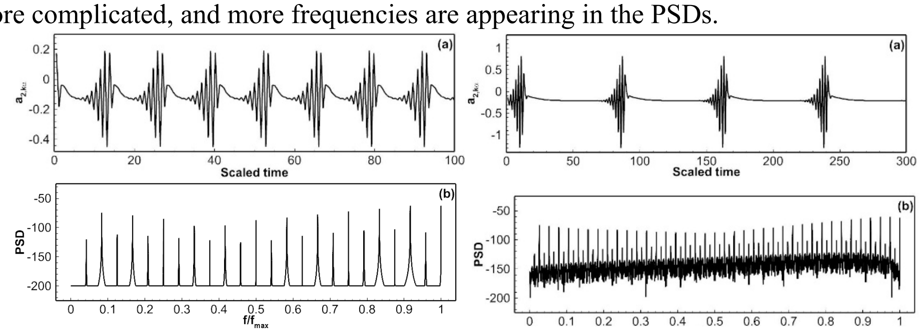
3) Construct the PMLB equation through a Galerkin procedure

$$
\begin{aligned}
a_{i,\vec{k}}(t+\delta t) = {} & \left(1-\delta t\vec{e}_i\cdot\vec{k}-\tfrac{1}{\tau}\right)a_{i,\vec{k}}(t)+\tfrac{1}{\tau}\left\{\omega_i\textstyle\sum_{j=0}^{18}a_{j,\vec{k}}(t)+\frac{3\omega_i\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)}{c^2}\right\}+ \\
& \tfrac{1-\beta}{\tau}\Bigg\{\frac{9\omega_i}{2\sum_{j=0}^{18}\tilde{f}_j c^4}\left[2\left(\vec{e}_i\cdot\textstyle\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\right)\left(\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\right)+\left(\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\right)^2\right]- \\
& \frac{9\omega_i}{2\left(\sum_{j=0}^{18}\tilde{f}_j\right)^2c^4}\left[\textstyle\sum_{j=0}^{18}a_{j,\vec{k}}(t)\left(\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\right)^2+2\sum_{j=0}^{18}a_{j,\vec{k}}(t)\left(\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\right)\left(\vec{e}_i\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\right)\right]- \\
& \frac{3\omega_i}{2\sum_{j=0}^{18}\tilde{f}_j c^2}\left(\textstyle\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)+\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\right)+ \\
& \frac{3\omega_i}{2\left(\sum_{j=0}^{18}\tilde{f}_j\right)^2c^2}\left(\textstyle\sum_{j=0}^{18}a_{j,\vec{k}}(t)\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\cdot\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j+\sum_{j=0}^{18}a_{j,\vec{k}}(t)\sum_{j=0}^{18}\vec{e}_j\tilde{f}_j\cdot\sum_{j=0}^{18}\vec{e}_j a_{j,\vec{k}}(t)\right)\Bigg\}.
\end{aligned}
$$

The four bifurcation parameters are the three components of the wavevector $\vec{k}$ and the relaxation time τ.

3 Results and Future Work

Numerical results for two representative behaviors—subharmonic (left) and noisy subharmonic (right) —for laminar and turbulent flow respectively, are shown below. The top and bottom rows are for the time series of $a_{i,\vec{k}}$ and the corresponding power spectral density (PSD), respectively. For the laminar subharmonic (left), the bottom of the time series for adjacent periods are different and there are one-period doublings in the corresponding PSD. For the turbulent subharmonic behavior (right), the time series are more complicated, and more frequencies are appearing in the PSDs.



Besides the two behaviors above, the DDS captures laminar and turbulent behaviors from periodic, n-period, quasiperiodic, to noisy periodic with harmonics, noisy quasiperiodic, and broadband. These results imply the possibility of applying the DDS in a large-eddy simulation (LES) model to predict the SGS physics. In the next step, we will investigate the effects of the bifurcation parameters beyond the basic description provided in the current work. Also, it will then be possible to build the complete LES model based on LBM and validate its performance against physical measurements and direct numerical simulation results for problems of interest.

References

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