A Conservative Overset Method for Unstructured Grids

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I. Introduction & Approach

Chimera overset techniques for Computational Fluid Dynamics (CFD) are an effective way to model flows involving solid bodies which undergo large scale motions throughout the simulation. Overset methods involve solving the governing equations on multiple disparate meshes and then interpolating/communicating data between the meshes at the overlapping grid points. Current approaches, however, do not take into account the "double counting" which occurs in the overlapped regions on each of the meshes as demonstrated in Figure 1a. Therefore, conservation error is introduced into the simulation.

One method, which was developed Galbraith[1] and extended by Crabill[2] and Duan,[3] addressed this by following an abutting grid approach where a minimal overlap is created and the fluxes at the open faces of the abutting cells are exchanged between the mesh systems. Although this approach is not fully conservative (double counting of the overlap region still remains), it has been shown that optimization of the abutting face locations and solution interpolation can yield a conservation error that is the of the order of discretization error.

The goal of this current work is to develop a method that enhances the abutting grid approach while still being fully conservative. This new approach is described in Figure 1b. The overall goal of the conservative overset method, is to modify both the volume and flux integrals of both fringe elements such that they each only account for a portion of the overlap region. By doing so, the elements are no longer double counting the overlap region and are analagous to a single, fully connected grid. Note that this can be done for both Finite Volume (FV) and Finite Element (FE) methods.

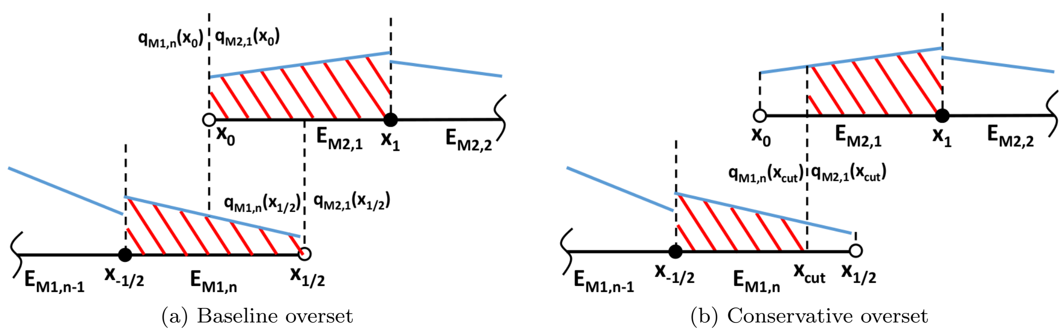

Figure 1: 1D schematic comparing the (a) baseline and (b) conservative overset DG methods. Volume integrals are computed over the striped red regions and fluxes from neighboring meshes are interpolated at (a) the element endpoints for the baseline method and (b) at the cut location for the conservative method

Using this approach, both the fluxes and the volume integrals are altered. The fluxes are handled similarly to the baseline approach however instead of interpolating the neighboring flux at the end point, now it is interpolated at the point where the overlapping region is cut, called x_{cut} in Figure 1b.

The volume integrals, which are only implemented in the FE method, are modified by subtracting the integral of the cut region, as shown in Equation (1). In order to compute the integral over the cut region, the same number of quadrature points are used to compute both integrals, maintaining the order of accuracy for the integration. The primary variables are computed at the quadrature points of the subcell using the traditional FE technique of interpolation using the element shape functions. This modification is done for volume integrals including both the mass matrix and the convective terms.

$$\int_a^b f(x)dx = \int_a^c f(x)dx - \int_b^c f(x)dx \quad \text{where } a < b < c \tag{1}$$

II. Preliminary Results

Preliminary results are discussed for a 1-D advection equation for a wave passing through two meshes on a periodic domain using Discontinuous Galerkin Finite Elements (DGFE). For this problem, theoretically after a single flow through after the wave returns to its original position, the final solution should be identical to the initial condition. The L2 and conservation error are shown in Figure 2. Both the baseline and conservative overset methods show convergence that follows the expected rate of $(p+1)$. Across the board the conservative method shows better convergence than the baseline methods. In terms of the conservation error, the baseline approach shows that the conservation error converges at roughly a rate of (p+1). On the other hand, the conservative approach shows conservation error on the order of machine zero in most cases.

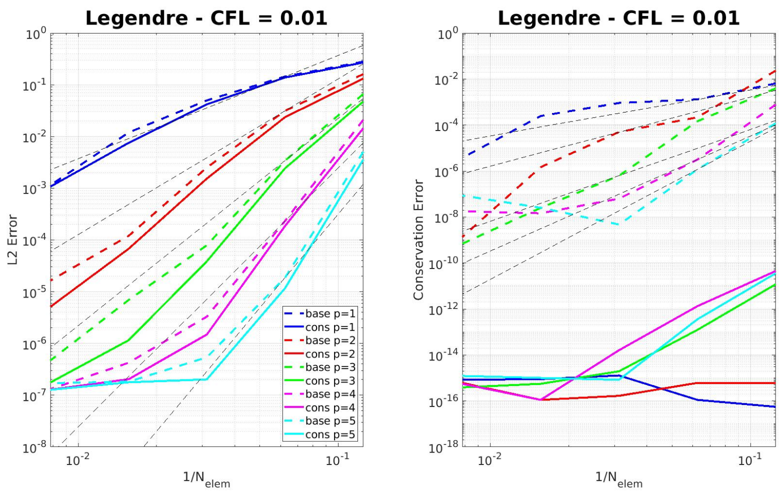

Figure 2: Convergence plots for L2 and conservation error using the baseline and conservative overset methods

III. Final Paper

The final paper will present the full details for the new conserative overset methods including:

- 1D implementation of the conservative overset method for both the Finite Volume (FV) and Discontinuous Galerkin Finite Element (DGFE) methods
- 2D implementation of the conservative overset method for DGFE
- Analysis of the stability of the conservative overset method as the cell cutting method is varied

References

[1]Marshall Galbraith, Paul Orkwis, and John Benek. Extending the discontinuous galerkin scheme to the chimera overset method. In *20th AIAA Computational Fluid Dynamics Conference*, page 3409, 2011.

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[3]Zhaowen Duan and Zhi J Wang. A high order overset fr/cpr method for dynamic moving grids. In *AIAA Scitech 2019 Forum*, page 1399, 2019.