A high-order low-dissipation Euler–Lagrange method for compressible gas-particle flows

Meet Patel*, Jesse Capecelatro**
Corresponding author: meetm@umich.edu

* Research Assistant, University of Michigan, Ann Arbor, Michigan, 48109.
** Assistant Professor, University of Michigan, Ann Arbor, Michigan, 48109.

Abstract: We present a low-dissipation strategy for simulating gas-particle compressible flows from dilute to dense concentrations. The volume-filtered compressible Navier–Stokes equations are discretized using high-order energy stable finite difference operators with localized shock capturing. Particle are tracked individually in a Lagrangian manner. The framework is applied to a three-dimensional simulation of an underexpanded jet impinging on a granular bed under varying nozzle pressure ratios. *NASA Award No. 80NSSC20K0295

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Particle-laden compressible flows are common to many environmental and engineering systems, such as volcanic eruptions, coal dust explosions, pulsed detonation engines, and plume-surface interactions during powered descent of a spacecraft. While much progress has been made in simulating incompressible particle-laden flows in recent decades, far less attention has paid to two-phase flows at finite Mach numbers. In this work, we present a high-order Eulerian–Lagrangian framework tailored for compressible turbulent flows laden with solid particles.

Applying a volume filter to the viscous compressible Navier–Stokes equations (excluding the volume occupied by particles) [3], the gas-phase equations can be expressed compactly as

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial}{\partial x_i}\left[\alpha\left(\boldsymbol{F}_i^I - \boldsymbol{F}_i^V\right)\right] = \boldsymbol{S}, \tag{1}$$

where $\boldsymbol{Q} = [\alpha\rho, \alpha\rho u_i, \alpha\rho E]^T$ is the vector of conserved variables, $\boldsymbol{F}^V$ and $\boldsymbol{F}^I$ are the viscous and inviscid fluxes, and $\boldsymbol{S}$ contains source terms that account for two-way coupling, given by

$$\boldsymbol{F}_i^I = \begin{bmatrix} \rho u_i \\ \rho u_1 u_i + p\delta_{i1} \\ \rho u_2 u_i + p\delta_{i2} \\ \rho u_3 u_i + p\delta_{i3} \\ u_i(\rho E + p) \end{bmatrix}, \quad \boldsymbol{F}_i^V = \begin{bmatrix} 0 \\ \tau_{1i} \\ \tau_{2i} \\ \tau_{3i} \\ u_j\tau_{ij} - q_i \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} 0 \\ (p\delta_{i1} - \tau_{i1})\frac{\partial\alpha}{\partial x_i} + \mathcal{F}_1 + g_1 \\ (p\delta_{i2} - \tau_{i2})\frac{\partial\alpha}{\partial x_i} + \mathcal{F}_2 + g_2 \\ (p\delta_{i3} - \tau_{i3})\frac{\partial\alpha}{\partial x_i} + \mathcal{F}_3 + g_3 \\ (\tau_{ij} - p\delta_{ij})\frac{\partial}{\partial x_i}(\alpha_p u_{p,j}) + q_i\frac{\partial\alpha}{\partial x_i} + u_{p,i}\mathcal{F}_i + \mathcal{Q} \end{bmatrix}.$$

The conserved variables include the gas-phase volume fraction α, density ρ, velocity u_i (in direction i), and total energy E. Pressure $p = (\gamma - 1)(\rho E - \rho u_i u_i/2)$, where $\gamma = 1.4$ is the ratio of specific heats, τ_{ij} is the viscous stress tensor, q_i is the heat flux, and g_i is gravity. Finally, $\mathcal{F}$ and $\mathcal{Q}$ are interphase momentum and heat exchange terms, respectively. The equations are discretized using a class of high-order finite difference operators that satisfy the summation-by-parts property. Boundary conditions are enforced weakly via the simultaneous approximation term (SAT) treatment. The combined SBP–SAT formulation provides an energy estimate [4]. Kinetic energy preservation is achieved using a skew-symmetric-type splitting of the inviscid fluxes [2] (extended here to account for α), which provides nonlinear stability at low Mach number. Shock capturing is handled via localized artificial dissipation.

The motion of each particle is calculated using Newton's second law of motion according to

$$\frac{\mathrm{d}\boldsymbol{x}_p^{(i)}}{\mathrm{d}t} = \boldsymbol{v}_p^{(i)} \quad \text{and} \quad m_p \frac{\mathrm{d}\boldsymbol{v}_p^{(i)}}{\mathrm{d}t} = V_p \nabla \cdot (\boldsymbol{\tau} - p\mathbf{I}) + \boldsymbol{F}_{\text{drag}}^{(i)} + \boldsymbol{F}_{\text{lift}}^{(i)} + \boldsymbol{F}_{\text{col}}^{(i)} + m_p \boldsymbol{g}, \tag{2}$$

where, $\boldsymbol{x}_p^{(i)}$ and $\boldsymbol{v}_p^{(i)}$ are the position and velocity of particle i, respectively, m_p is the mass of the particle and V_p is its volume. $\boldsymbol{F}_{\text{drag}}^{(i)}$, $\boldsymbol{F}_{\text{lift}}^{(i)}$, and $\boldsymbol{F}_{\text{col}}^{(i)}$ are force contributions due to drag, lift, and collisions, respectively, the latter accounted for via a soft-sphere collision model. Both phases are advanced in time simultaneously using the standard fourth-order Runge–Kutta scheme.

The framework is applied to a three-dimensional simulation of an under-expanded jet impinging on a bed of 44M particles (see Fig. 1). The domain is discretized on a Cartesian grid of size $247 \times 883 \times 883$. The nozzle of diameter D is modeled using a ghost-point immersed boundary method [1] and placed $3.75D$ above the bed. Two nozzle pressure ratios (NPR) are considered. The higher NPR results in the generation of a Mach disk that produces a qualitatively different crater. Future work will be conducted to provide quantitative comparisons and perform a modal analysis to understand the relationship between NPR and crater morphology.

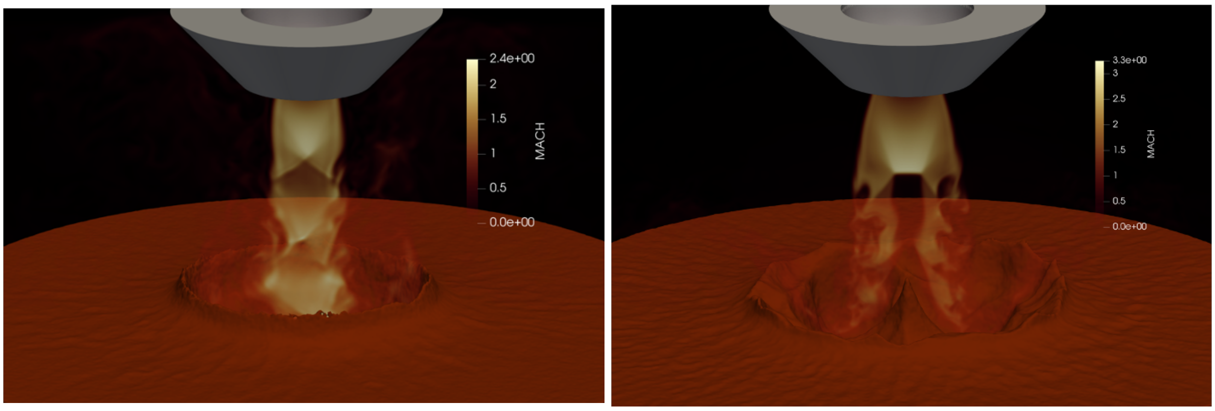

Figure 1: NPR= 4.08 (left). NPR= 6.12 (right).

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