# Output-Based $h$-$p$ Refinement Strategy with Anisotropic AMR and High-Order CENO Finite-Volume Scheme for Three-Dimensional Inviscid Flows

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**Abstract:** The formulation and application a hybrid $h$-$p$ refinement technique method based on output-based error estimation is considered for the prediction of compressible three-dimensional inviscid flows. The proposed $h$-$p$ refinement strategy makes use of a block-based anisotropic adaptive mesh refinement scheme (AMR) used in combination with a high-order CENO finite-volume spatial discretization scheme based on high-order solution reconstruction. The resulting scheme allows for refinement of the spatial discreatization procedure, in which either enhancements of the local mesh spacing, $h$, or increases in the order of solution reconstruction, $p$, are allowed to achieve the desired increased solution accuracy. Functional convergence rates and potential benefits of the proposed output-based approach are examined and compared to those of standard gradient-based methods for a range of compressible inviscid flow problems of varying complexity.

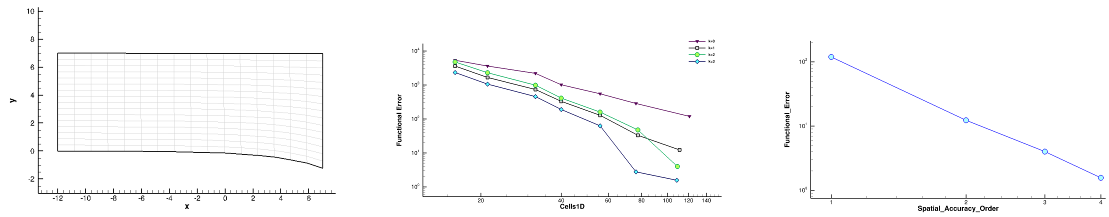
*Keywords:* Output-Based Error Estimation, Computational Fluid Dynamics.

## 1 Introduction and Motivation

High-order methods and adaptive mesh refinement (AMR) are two common numerical strategies employed to reduce computational costs of predicting complex physical flows having disparate spatial scales. Additionally, when used in conjunction with output-based error estimation methods, the mesh spacing, $h$, or order of the accuracy of the spatial discretization scheme, $p$, can be adapted within regions of the computational domain having the largest contributions to errors associated with a specified quantity of interest ensuring the functional errors are controlled and reduced. Both $h$-based (mesh adaptation with fixed discretization accuracy) and $p$-based output-based methods (order of accuracy adaptation with a fixed mesh) have been proposed and considered in previous studies. While it has proven difficult to achieve for practical implementations, combined $h-p$ refinement strategies have the potential to yield improved convergence characteristics when compared to pure $h$- or $p$-refinement methods alone.

## 2 Scope and Preliminary Results

With this viewpoint in mind, a novel hybrid *h*-*p* refinement technique method based on output-based error estimation is proposed and applied to the prediction of compressible three-dimensional (3D) inviscid flows. The proposed *h*-*p* refinement strategy makes use of the anisotropic AMR of Freret and Groth [1], valid for 3D, multi-block, body-fitted, hexahedral meshes, and is used in combination with the high-order central essentially non-oscillatory (CENO) finite-volume spatial discretization scheme of Ivan and Groth [2]. The latter is based on high-order reconstruction in which the spatial discretization scheme is applied to the solution of the integral form of the Euler equations governing compressible 3D inviscid gaseous flows for each hexahedral cell of the computational mesh. Newton's method is applied to the solution of the coupled non-linear algebraic equations to obtain steady-state solution to the governing equations on a given computational grid. A linear system of equations is solved resulting from the Jacobian of the steady-state solution residual and derivatives of a defined functional of interest to yield values of the discrete adjoint solution and functional error estimates and refinement indicators are calculated to identify regions for increased solution accuracy [3]. Additionally, a measure of solution smoothness is used to select between *h*- and *p*- refinement: local *h*-refinement is applied in non-smooth regions while local *p*-refinement is applied where the flow is smooth.




**(a)** Initial mesh consisting of 1 block (1024 cells).

**(b)** Adaptive *h*-refinement.

**(c)** *p*-refinement.

**Figure 1:** Effect on adaptive *h*-refinement and uniform *h*-refinement on varying orders of spatial discretization, and uniform *p*-refinement.

Preliminary results obtained using the proposed output-based *h*-*p* refinement method are depicted in Figure 1 for supersonic inviscid flow of air past a smooth corner at Mach 2.5 at $\rho$=1.225 kg/$m^3$ and p=101,325 Pa. The initial mesh for the two-dimensional domain consisted of a single grid block with 1,024 computaitonal cells is given in Figure 1(a). Numerical results for the convergence of the functional error obtained using *h*- and *p*-refinement procedures alone are given in Figure 1(b) and Figure 1(c), respectively, demonstrating the reduction of the functional error with both increased mesh resolution as well as with increases in the order of the spatial reconstruction. The final version of the paper will include results for the hybrid *h*-*p* method with both combined *h*- and *p*-refinement with application to a range of 3D inviscid flows.

# References

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