A Nonlinear Schur Complement Solver for CFD-Based Multidisciplinary Models

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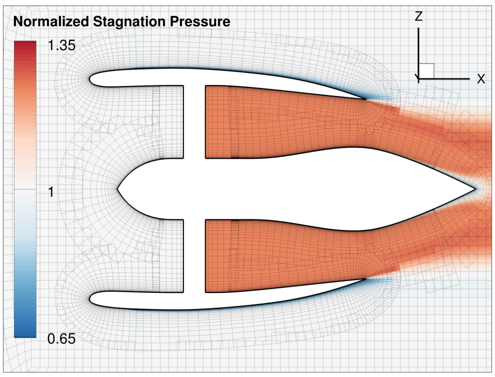
Abstract: CFD-based multidisciplinary models are the fundamental building blocks of multidisciplinary design optimization frameworks. Linear and nonlinear solutions of these coupled models are difficult, especially when the Jacobian matrices represent a saddle point problem, where a block-diagonal corresponding to a discipline is non-invertible. These scenarios necessitate the use of a coupled solver algorithm such as the Newton's method instead of the popular block Gauss–Seidel-based methods because of this non-invertible block. To address this challenge, we introduce a nonlinear Schur complement solver suitable for CFD-based multidisciplinary models. The solver leverages the specialized linear and nonlinear solvers of the CFD code, and therefore, does not require the solution of a large coupled linear system as the coupled Newton's method. Furthermore, because the solver primarily uses the specialized linear and nonlinear solvers of the CFD code, it does not suffer from the same robustness limitations as the coupled Newton's method. In this work, we will implement this solver in NASA's OpenMDAO framework and demonstrate its effectiveness using a CFD-based aeropropulsive model.

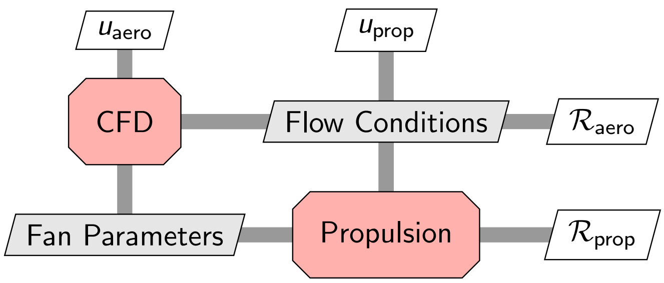
1 Introduction

To reduce the environmental impact of aviation, new aircraft concepts rely on advanced technologies such as boundary layer ingestion or distributed electric propulsion. However, we do not have any prior experience in the design of these systems, and therefore, we must rely on multidisciplinary design optimization (MDO) frameworks to maximize the interdisciplinary benefits of these technologies [1].

Solving the linear and nonlinear systems of equations that arise from multidisciplinary models is challenging. The most common solution approaches for these problems are based on the linear and nonlinear block Gauss–Seidel methods. However, there is a class of multidisciplinary models where Gauss–Seidel-based methods cannot be used. To introduce this class of problems, we will use an aeropropulsive model from previous work as an example (Fig. 1) [2].



(a) Contours of normalized stagnation pressure for the podded fan model.



(b) Simplified XDSM diagram of the coupled aeropropulsive model.

Figure 1: CFD solution and the XDSM diagram of the podded fan aeropropulsive model [2].

This model results in a coupled nonlinear system of equations, which can be written as:

$$\mathcal{R}(u) = \begin{bmatrix} \mathcal{R}_{\text{aero}}(u_{\text{aero}}, u_{\text{prop}}) \\ \mathcal{R}_{\text{prop}}(u_{\text{aero}}, u_{\text{prop}}) \end{bmatrix} = 0, \tag{1}$$

where $\mathcal{R}_{\text{aero}}$ and $\mathcal{R}_{\text{prop}}$ are the residual functions for the aerodynamic and propulsion models, respectively. In the general case, both of these residuals depend on both the aerodynamic (u_{aero}) and propulsion (u_{prop}) models' states; however, for this aeropropulsive model that uses boundary conditions to model the effect of the fan on the flow, the propulsion model residuals ($\mathcal{R}_{\text{prop}}$) only depend on the aerodynamic model's states (u_{aero}). As a result, the block diagonal term $\partial\mathcal{R}_{\text{prop}}/\partial u_{\text{prop}}$ term on the Jacobian matrix is zero:

$$\frac{\partial \mathcal{R}}{\partial u} = \begin{bmatrix} \partial\mathcal{R}_{\text{aero}}/\partial u_{\text{aero}} & \partial\mathcal{R}_{\text{aero}}/\partial u_{\text{prop}} \\ \partial\mathcal{R}_{\text{prop}}/\partial u_{\text{aero}} & \partial\mathcal{R}_{\text{prop}}/\partial u_{\text{prop}} \end{bmatrix} = \begin{bmatrix} \partial\mathcal{R}_{\text{aero}}/\partial u_{\text{aero}} & \partial\mathcal{R}_{\text{aero}}/\partial u_{\text{prop}} \\ \partial\mathcal{R}_{\text{prop}}/\partial u_{\text{aero}} & 0 \end{bmatrix}. \tag{2}$$

This feature of the Jacobian matrix prevents the use of the linear and nonlinear block Gauss–Seidel methods in the linear and nonlinear solutions of this saddle point problem [3].

In previous work [2], we addressed the challenge of converging the propulsion model residuals by adding them as equality constraints to the optimization problem, which avoids the requirement of solving them at each model evaluation. This approach is not ideal, because we cannot perform standalone analyses using this model, and any balanced analysis requires the solution of a small optimization problem. An alternative solution here would be to use the coupled Newton's method to solve the nonlinear system; however, forming and solving a large linear system for the Newton's method is difficult, and the method suffers from robustness issues when the initial guess is far away from the solution. The ideal solution here would achieve Newton-like convergence rates while relying on the specialized linear and nonlinear solvers of the CFD solver as much as possible.

2 Proposed Work

In this work, we will develop a nonlinear solver based on the Schur complement method to solve the saddle point problems arising from multidisciplinary models. To achieve this, we will combine the ideas from the approximate Newton–Krylov solvers [4] and nonlinear preconditioning approaches [5]. As a result, this approach will utilize the specialized nonlinear and linear solvers of the CFD code and does not require forming and solving a larger linear system.

We will implement this solver in NASA's OpenMDAO framework [6] and demonstrate its capabilities with the aeropropulsive design optimization problem from our previous work [2]. This solver will enable robust analyses of CFD-based multidisciplinary problems that exhibit saddle point properties and will accelerate future design optimization studies that rely on CFD-based multidisciplinary models.

References

[1] Martins, J. R. R. A., and Ning, A., *Engineering Design Optimization*, Cambridge University Press, 2022. doi:10.1017/9781108980647, URL `https://mdobook.github.io`.

[2] Yildirim, A., Gray, J. S., Mader, C. A., and Martins, J. R. R. A., "Coupled Aeropropulsive Design Optimization of a Podded Electric Propulsor," *AIAA Aviation Forum*, 2021. doi:10.2514/6.2021-3032.

[3] Benzi, M., Golub, G. H., and Liesen, J., "Numerical Solution of Saddle Point Problems," *Acta Numerica*, Vol. 14, 2005, pp. 1–137. doi:10.1017/S0962492904000212.

[4] Yildirim, A., Kenway, G. K. W., Mader, C. A., and Martins, J. R. R. A., "A Jacobian-free approximate Newton–Krylov startup strategy for RANS simulations," *Journal of Computational Physics*, Vol. 397, 2019, p. 108741. doi:10.1016/j.jcp.2019.06.018.

[5] Dolean, V., Gander, M. J., Kheriji, W., Kwok, F., and Masson, R., "Nonlinear Preconditioning: How to Use a Nonlinear Schwarz Method to Precondition Newton's Method," *SIAM Journal on Scientific Computing*, Vol. 38, No. 6, 2016, pp. A3357–A3380. doi:10.1137/15M102887X.

[6] Gray, J. S., Hwang, J. T., Martins, J. R. R. A., Moore, K. T., and Naylor, B. A., "OpenMDAO: An open-source framework for multidisciplinary design, analysis, and optimization," *Structural and Multidisciplinary Optimization*, Vol. 59, No. 4, 2019, pp. 1075–1104. doi:10.1007/s00158-019-02211-z.