Deneb: An Open-source High-performance Flow Solver based on DRM-DG Method

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Abstract: In efforts to develop efficient and accurate higher-order numerical methods and a flow solver for scientific computing and industrial applications, Deneb—an open-source high-performance flow solver—is developed based on the direct reconstruction method for discontinuous Galerkin (DRM-DG) method. Deneb is aimed at multi-physical flow simulations as well as high-fidelity scale-resolving simulations. The performance of the developed solver is thoroughly verified and validated through a class of benchmark problems ranging from inviscid supersonic flows to hypersonic equilibrium flows and magneto-hydrodynamic (MHD) flows.

Keywords: Direct Reconstruction Method, Discontinuous Galerkin Method, Hypersonic Flow, Magneto-hydrodynamic Flow, Open-source Code.

1 Introduction

During the last few decades, finite element-based high-order methods for computational fluid dynamics (CFD) have been rapidly developed due to the remarkable potential and benefits for simulating complicated turbulent flows in scientific computing and industrial applications. Among several methods available, the physical domain-based discontinuous Galerkin (DG) method is chosen because of its flexibility to handle arbitrary polyhedral meshes without the order degradation on non-affine mapped elements. It, however, suffers a huge increase in computational costs (*i.e.*, computational time and memory overhead) when a nonlinear mapping from the physical to reference domain is introduced on a curved element. To overcome this, the direct reconstruction method (DRM) has been proposed and applied to the DG approximation in the previous studies [1, 2, 3, 4], leading to the DRM-DG formulation. The DRM-DG method provides a significant performance improvement compared to the quadrature-based DG method without compromising numerical accuracy. In this work, we present the Deneb—an open-source high-performance flow solver—that has been developed based on the high-order DRM-DG method with aiming at simulations of multi-physical flows and multi-scale turbulent flows.

2 Direct Reconstruction Method for Discontinuous Galerkin Approximation (DRM-DG)

The weak formulation of the DG method for a hyperbolic conservation law, $\partial q/\partial t + \boldsymbol{\nabla} \cdot \mathbf{F} = 0$, on a non-overlapping element D is given by

$$\int_D \frac{\partial q_h}{\partial t} \phi dV + \sum_{e \in \partial D} \int_e \phi \widehat{\mathbf{F}} \cdot \mathbf{n} dA - \int_D \mathbf{F} \cdot \boldsymbol{\nabla} \phi dV = 0. \tag{1}$$

In DRM, both surface and volume fluxes in Eq. (1) are interpolated by the generalized reconstruction directly in the physical coordinate system [1, 2]. Using the generalized reconstruction, the surface and volume integral functions in Eq. (1) are re-expressed as

$$\int_e \phi \widehat{\mathbf{F}} \cdot \mathbf{n} dA \approx \sum_{i=1}^{\dim \mathbb{T}_1(D)|_e} \widehat{\mathbf{F}}|_{\mathbf{x}_i^e} \cdot \int_e \phi \ell_i^e(\mathbf{x}) \mathbf{n} dA, \tag{2a}$$

$$\int_D \mathbf{F} \cdot \boldsymbol{\nabla} \phi dV \approx \sum_{i=1}^{\dim \mathbb{T}_2(D)} \mathbf{F}|_{\mathbf{x}_i} \cdot \int_D \ell_i(\mathbf{x}) \boldsymbol{\nabla} \phi dV. \tag{2b}$$

One can freely choose the target spaces, $\mathbb{T}_1$ and $\mathbb{T}_2$, and the reconstruction nodes, $\mathbf{x}_i^e$ and $\mathbf{x}_i$. The reconstruction functions, $\ell_i^e(\mathbf{x})$ and $\ell_i(\mathbf{x})$, are computed from the generalized inversion of a generalized Vandermonde matrix. The DRM-DG method is finally achieved by plugging Eq. (2) into Eq. (1).

3 Solver Description

Deneb is an open-source high-performance flow solver based on the high-order DRM-DG method to conduct multi-physical flow simulations as well as high-fidelity scale-resolving simulations. Exploiting the object-oriented programming (OOP) paradigm, it utilizes C++ class structures for maximizing extendability, manageability, and maintainability. Utilizing MPI (Message Passing Interface) and METIS/ParMETIS libraries, Deneb is capable of performing large-scale parallel computations on a distributed memory-based cluster. Tensor operations are efficiently computed by the GEMM-based approach on Intel MKL (Math Kernel Library) [3].

Detailed description of numerical methods used and extensive computations for verification and validation will be provided in the full manuscript.

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