Assessment of a high-order implicit residual smoothing time scheme for multiblock curvilinear meshes

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In direct and large eddy simulations, very small space steps are used close to the solid walls in order to resolve the boundary-layer structures. Due to the restrictive CFL stability criteria of explicit time-stepping schemes, the maximum allowable time step is also very small, leading to high computational costs, notably for converging flow statistics. The use of an implicit integration scheme may overcome this limitation at the price of an increased computional cost per step. Furthermore, the most commonly used fully implicit schemes induce higher errors due to the necessary approximations and bad dispersion and dissipation properties. As a compromise, a fourth-order implicit residual smoothing scheme (IRS4), succesfully validated for a finite volume solver in [1, 2], has been implemented in a multiblock high-order finite-difference solver. For moderate CFL numbers, a similar accuracy as the explicit method is obtained with substantial savings in terms of computational time.

Extension of 4^{th}-order IRS scheme to curvilinear grids

The code solves the compressible Navier-Stokes equations for multiblock geometries, using a coordinate transform for curvilinear grids. The inviscid fluxes are discretized with 10th-order standard centred differences whereas 4th-order is used for viscous fluxes. A 9th-order selective filtering or artificial dissipation is used to eliminate grid-to-grid oscillations. The explicit time advancement is realised with a low-storage four-stage Runge-Kutta (RK4). For the transformation $(x, y, z) \rightarrow (\xi, \eta, z)$, the IRS4 operator is applied at each Runge-Kutta stage as:

$$\left\{\begin{array}{l} U^{(0)} = U^n \\ \mathcal{J}\Delta U^{(k)} = -a_k \Delta t \mathcal{R}(U^{(k-1)}), \quad k = 1, ..., s \\ U^{n+1} = U^{(s)} \end{array}\right. \qquad \text{with} \quad \mathcal{J} = \prod_{l=1}^{d} [1 + \theta_4 (\tfrac{\Delta t}{\delta x_l})^4 \delta_l ({\lambda_l^e}^4 \delta_l^3)] \tag{1}$$

where $\mathcal{J}$ denotes the implicit operator, θ_4 a tuning parameter determined to ensure stability [1], δx_l the space step ($\delta\xi = \delta\eta = 1$) and λ_l^e the spectral radius of the flux Jacobian in the l^{th} direction. For curvilinear coordinates, the spectral radius is set as $\lambda_\xi^e = \lambda_\eta^e = \sqrt{\theta_\xi^2 + \theta_\eta^2} + c\sqrt{||\nabla\xi||_2^2 + ||\nabla\eta||_2^2}$, with θ_ξ and θ_η the contravariant velocities. A particular attention has been paid to the boundary treatment, notably at the wall and at the interfaces of blocks or MPI domains. The IRS4 operator $\mathcal{J}$ being a bilaplacian filter, a pentadiagonal linear system is inverted for each implicited direction. Solving the global linear system distributed over the processors would be too costly, since direct pentadiagonal solvers are difficult to parallelize. That is why local inversions are performed using 5 rows of ghost points (for the 11-pts stencil scheme in use). Close to physical boundaries or domain interfaces, the implicit operator is degraded into IRS1 [3] at the first point and IRS2 at the second point:

$$\left\{\begin{array}{ll} \mathcal{J} = 1 - \theta_1 \frac{\Delta t}{\delta x_l} \delta^+ (\lambda_l^e) & \text{if the boundary is at the left} \\ \mathcal{J} = 1 + \theta_1 \frac{\Delta t}{\delta x_l} \delta^- (\lambda^{e_l}) & \text{if the boundary is at the right} \\ \mathcal{J} = 1 - \theta_2 (\frac{\Delta t}{\delta x})^2 \delta ({\lambda^e}^2 \delta) & \text{for the second point} \end{array}\right. \tag{2}$$

with δ^+ the upward difference operator and δ^- the backward difference IRS1 operator. Furthermore, the solution increment is set to 0 at the wall. However, the simplified treatment at interfaces may introduce numerical instabilities. To quantify the effect of boundary treatments, a stability study for a 1-D linear advection problem has been carried out and will be presented. To avoid approximations at MPI interfaces, a full parallelization of the linear solver would be necessary. Solutions based on the divide-and-conquer algorithm, such as the banded solver of ScaLapack, has been tested but leads to an unacceptable overcost. Another possibility would be to extend more efficient tridiagonal parallel solvers, such as Pascal library [4], to pentadiagonal systems. Some approximations have still to be introduced in order to reduce the overcost.

Preliminary results and work in progress

Channel flow: The first case investigated is the turbulent channel flow at $Re_\tau = 180$, based on the friction velocity and the channel half-height H. The computational domain of $4\pi H \times 2H \times 2\pi H$ is discretized with $192 \times 180 \times 160$, leading to a DNS resolution ($\Delta x^+ = 11.9$, $\Delta z^+ = 7.1$, $\Delta y_w^+ = 0.8$ and $\Delta y_c^+ = 4$). Isothermal

Case	Δt^+	$\frac{t_{CPU,expl.}}{t_{CPU,case}}$
Explicit	1.05×10^{-2}	1.0
IRS4	5.26×10^{-2}	4.06

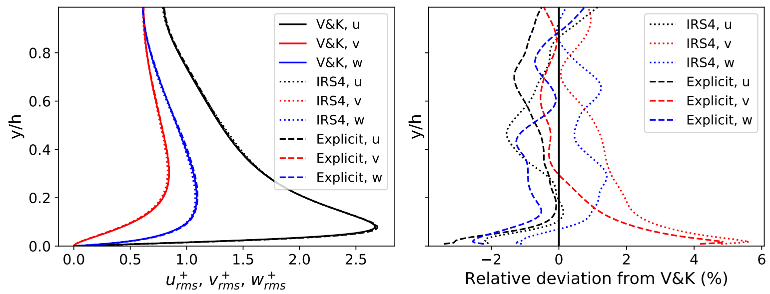
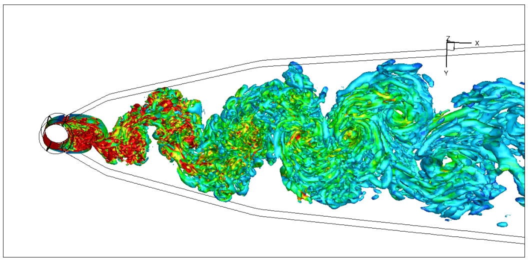
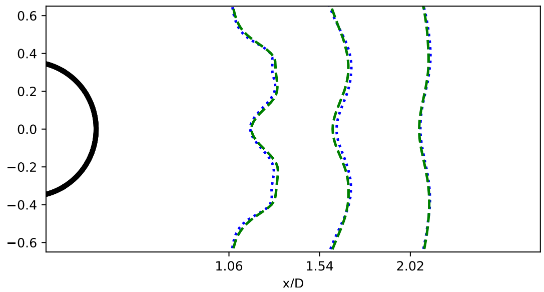


Figure 1: Comparison of the rms velocities profiles with the Vreman & Kuerten (V&K) reference [5] for the explicit and IRS4 time integrations with Δt^+_{IRS4}=5 $\times$ $\Delta t^+_{\text{explicit}}$.





Case	Δt	$\frac{t_{CPU,expl.}}{t_{CPU,case}}$
Explicit	4.91×10^{-9}	1.0
IRS4	14.74×10^{-9}	2.3

Figure 2: Left: Snapshot of λ_2 vorticity criterion. Right: comparison of the $\frac{\overline{u'u'}}{U_{in}}$ profiles in the wake of the cylinder at $\frac{x}{D}$ = 1.06, 1.54 and 2.02 for the 2 cases (······: Explicit, - - -: IRS4).

no-slip conditions are applied at the walls and periodicity conditions along streamwise and spanwise directions. The Mach number is set to $M = 0.3$. Results using an explicit time integration are compared with the use of IRS4 implicitation for various CFL numbers. Figure 1 shows a comparison between the explicit solution using a time step Δt^+ and IRS4 applied only in the wall-normal direction with $5\Delta t^+$. The two simulations are run during the same physical time interval. A saving of a factor 4 is obtained (see table) for a similar accuracy, as shown by the relative deviation with respect to the reference solution of [5] (Fig.1).

Cylinder flow: The IRS4 method is then validated for the flow past a circular cylinder at $Re_D = 3900$ based on the diameter D, and at $M = 0.3$, which is a common benchmark case for curvilinear geometries [6]. The simulation is performed on a multi-block H-O-H grid topology with approximatively 5 millions points ($\Delta y_w = 1.16 \times 10^{-6}$ m). Non-reflecting Tam & Dong's conditions are applied at free boundaries and a sponge zone is added at the outlet boundary. Explicit and implicit simulations are started from the same initial field with dimensional time steps given in the Table of Fig. 2. The IRS4 is applied in the ξ and η-directions with a timestep multiplied by 3. Time-averaged fields, computed over the same physical duration, are in good agreement, as shown for instance for streamwise rms velocity shown in the same figure. Small discrepancies are due to the limited averaging period allowed by the explicit scheme. The flow regime is chaotic at this Reynolds number and it would require very long averaging times to achieve well-converged statistics. The present implicitation leads a computational time reduction of a factor 2.3 (see table), so that longer time integrations are made possible.

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