Positivity-Preserving Entropy Stable Spectral Collocation Schemes of Arbitrary Order of Accuracy for the 3-D Navier-Stokes Equations

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Abstract: In this talk, we present a new family of positivity-preserving, entropy stable spectral collocation schemes of arbitrary order of accuracy for the 3-D compressible Navier-Stokes equations on unstructured hexahedral grids. To provide the positivity preservation and excellent discontinuity-capturing properties, the Navier-Stokes equations are regularized by adding an artificial dissipation in the form of the Brenner-Navier-Stokes diffusion operator. The resultant scheme is conservative, pointwise positivity preserving, design-order accurate for smooth solutions, and entropy stable, thus facilitating a rigorous L_2-stability proof for the symmetric form of the discretized Navier-Stokes equations. Numerical results demonstrating accuracy and positivity-preserving properties of the new schemes are presented for viscous flows with nearly vacuum regions and very strong shocks and contact discontinuities.

Keywords: Summation-by-parts operators, Entropy stability, Positivity, Brenner regularization, Navier-Stokes equations.

1 Methodology

The key distinctive feature of the proposed methodology is that it is proven to guarantee the pointwise positivity of thermodynamic variables for compressible viscous flows. The new schemes are constructed by combining a positivity-violating entropy stable method of arbitrary order of accuracy and a novel first-order positivity-preserving entropy stable method discretized on the same Legendre-Gauss-Lobatto collocation points used for the high-order counterpart.

The high-order entropy stable positivity-violating scheme is constructed by discretizing the 3-D Navier-Stokes equations by using the multidimensional tensor product summation-by-parts operators in each hexahedral grid element individually. The inviscid fluxes are approximated, such that they mimic the entropy compatibility condition $\boldsymbol{w}^T \frac{\partial f_m}{\partial x_m} = \frac{\partial \mathcal{F}_m}{\partial x_m}, m = 1,2,3$ at the discrete level, where $\boldsymbol{w}$ is a vector of entropy variables and $\mathcal{F}_m$ is an m-th component of the entropy flux. To provide excellent discontinuity-capturing properties, the Navier-Stokes equations are regularized by adding an artificial dissipation in the form of the Brenner-Navier-Stokes diffusion operator [1]. Furthermore, the viscous and artificial dissipation fluxes are constructed such that they are symmetrized by the entropy variables of the Navier-Stokes equations, thus facilitating the entropy stability proof for the 3-D discretized compressible Navier-Stokes equations. Since the high-order discrete differentiation operators do not in general satisfy the maximum principle, this numerical scheme is entropy stable, but not positivity preserving.

We overcome this problem by developing a new 1st-order scheme that guarantees the pointwise positivity of density and temperature and satisfies the entropy inequality. This first-order scheme is discretized on the same Legendre-Gauss-Lobatto points used for the high-order method and can be represented in the same form as the high-order scheme, where the high-order inviscid and artificial

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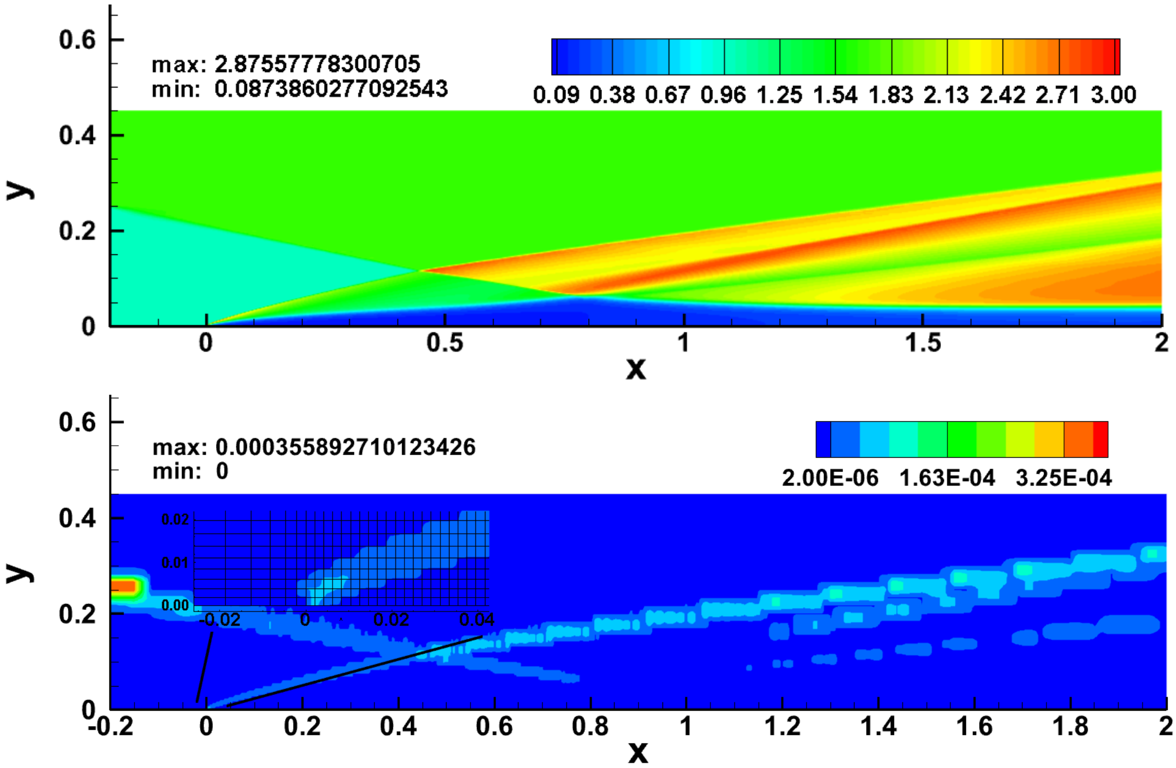

Figure 1.

dissipation fluxes are replaced with their low-order counterparts. To provide both the positivity of thermodynamic variables and entropy stability, the new first-order artificial dissipation operators are constructed by discretizing the viscous fluxes of the Brenner-Navier-Stokes equations in a special form. In contrast to the existing positivity-preserving schemes that rely on monotonicity properties of the Rusanov-type dissipation, the proposed method minimizes the amount of artificial dissipation required for pointwise positivity of density and temperature and uses novel entropy stable velocity and temperature limiters to eliminate the time step stiffness for viscous flows with strong discontinuities.

The high- and low-order schemes are combined by using a flux-limiting procedure, so that the resultant scheme guarantees pointwise positivity of thermodynamic variables, preserves design-order of accuracy for smooth solutions, and satisfies the discrete entropy inequality, thus facilitating a rigorous L_2-stability proof for the symmetric form of the discretized Navier-Stokes equations. Note that this flux-limiting procedure is used only in those elements where the solution loses its regularity.

2 Preliminary Results

A test problem demonstrating the performance of the new schemes is shown in Fig. 1 that presents density (top panel) and artificial viscosity contours computed with the new high-order ($p = 6$) positivity-preserving entropy stable spectral collocation scheme for the shock/boundary layer interaction flow at the Mach and Reynolds numbers set to M=6.85 and Re=10^5, respectively. In the final paper, we will present further numerical results demonstrating accuracy, entropy stability, and positivity-preserving properties of the new high-order schemes for viscous flows with nearly vacuum regions and very strong shocks and contact discontinuities.

References

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