Stable and non-dissipative kinetic-energy and entropy preserving (KEEP) schemes for compressible flows

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Abstract: Stable and non-dissipative kinetic-energy and entropy preserving (KEEP) schemes have recently been proposed for compressible flows. The KEEP schemes significantly improve numerical robustness compared to the kinetic energy preserving (KEP) schemes, which are widely recognized as non-dissipative and stable numerical schemes. The present study comprehensively discusses the KEEP schemes, including the derivation background and numerical formulations.

Keywords: Non-dissipative numerical schemes, Kinetic energy and entropy preservation, Split convective forms, Compressible flows.

1 Introduction

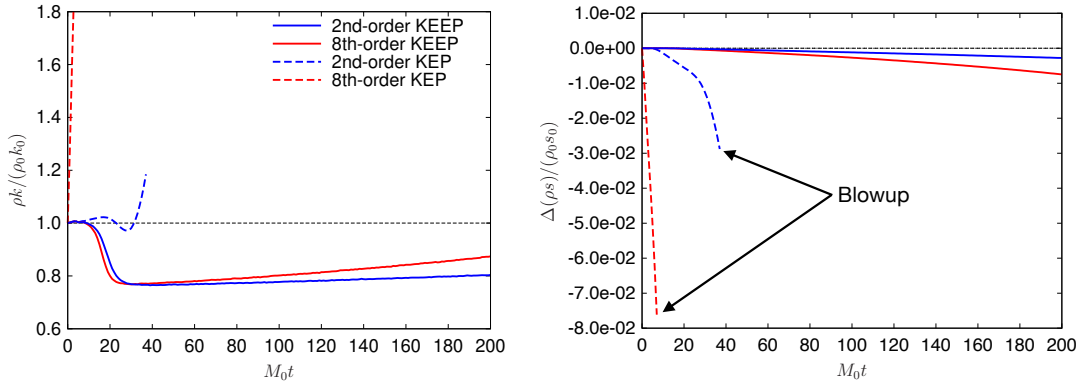
It is crucial to use non-dissipative numerical schemes to perform high-fidelity flow computations, such as direct numerical simulation and large eddy simulation (LES). Recently, we have proposed stable and non-dissipative kinetic-energy and entropy preserving (KEEP) schemes [1-3] for compressible flows. Although kinetic energy preserving (KEP) schemes [4] are widely recognized as non-dissipative and stable numerical schemes, the KEEP schemes significantly improve numerical robustness compared to the KEP schemes, particularly when compressible effects appear. In the KEEP schemes, the enhancement of numerical robustness is achieved by exactly preserving the kinetic energy and approximately preserving the entropy. A recent work [5] has succeeded in performing a wall-modeled LES of a full aircraft configuration using the KEEP schemes.

2 KEEP schemes and numerical test

The second-order accurate KEEP schemes for Cartesian grids were proposed first [1,2]. Then, the high-order accurate KEEP schemes in generalized curvilinear coordinates were also proposed [3]. For example, the second-order KEEP numerical fluxes proposed in Ref. [1] are

$$
\begin{aligned}
&\widetilde{\mathcal{C}_j}|_{(m\pm\frac{1}{2})} \equiv \frac{\rho|_{(m)}+\rho|_{(m\pm1)}}{2}\frac{u_j|_{(m)}+u_j|_{(m\pm1)}}{2}, \quad \widetilde{\mathcal{M}_{ij}}|_{(m\pm\frac{1}{2})} \equiv \widetilde{\mathcal{C}_j}|_{(m\pm\frac{1}{2})}\frac{u_i|_{(m)}+u_i|_{(m\pm1)}}{2}, \\
&\widetilde{\mathcal{K}_j}|_{(m\pm\frac{1}{2})} \equiv \widetilde{\mathcal{C}_j}|_{(m\pm\frac{1}{2})}\frac{u_i|_{(m)}u_i|_{(m\pm1)}}{2}, \quad \widetilde{\mathcal{I}_j}|_{(m\pm\frac{1}{2})} \equiv \widetilde{\mathcal{C}_j}|_{(m\pm\frac{1}{2})}\frac{e|_{(m)}+e|_{(m\pm1)}}{2}, \\
&\widetilde{\mathcal{G}_{ij}}|_{(m\pm\frac{1}{2})} \equiv \frac{(p\delta_{ij})|_{(m)}+(p\delta_{ij})|_{(m\pm1)}}{2}, \quad \widetilde{\mathcal{P}_j}|_{(m\pm\frac{1}{2})} \equiv \frac{u_j|_{(m)}p|_{(m\pm1)}+u_j|_{(m\pm1)}p|_{(m)}}{2},
\end{aligned}
\tag{1}
$$

where $\widetilde{\mathcal{C}}$, $\widetilde{\mathcal{M}}$, $\widetilde{\mathcal{K}}$, $\widetilde{\mathcal{I}}$, $\widetilde{\mathcal{G}}$, and $\widetilde{\mathcal{P}}$ are the numerical fluxes of the mass, momentum, kinetic energy, internal energy, pressure-gradient, and pressure-diffusion terms of the Euler equations, respec-



(a) Total kinetic energy (b) Total entropy

Figure 1: Time evolution of total kinetic energy and total entropy in inviscid Taylor-Green vortex test on a curvilinear grid at M_0=0.4.

tively:

$$\frac{\partial \rho u_j}{\partial x_j} \simeq \frac{\widetilde{\mathcal{C}}_j|_{(m+\frac{1}{2})} - \widetilde{\mathcal{C}}_j|_{(m-\frac{1}{2})}}{\Delta x_j}, \quad \frac{\partial \rho u_i u_j}{\partial x_j} \simeq \frac{\widetilde{\mathcal{M}}_{ij}|_{(m+\frac{1}{2})} - \widetilde{\mathcal{M}}_{ij}|_{(m-\frac{1}{2})}}{\Delta x_j}, \quad \frac{\partial \rho k u_j}{\partial x_j} \simeq \frac{\widetilde{\mathcal{K}}_j|_{(m+\frac{1}{2})} - \widetilde{\mathcal{K}}_j|_{(m-\frac{1}{2})}}{\Delta x_j},$$

$$\frac{\partial \rho e u_j}{\partial x_j} \simeq \frac{\widetilde{\mathcal{I}}_j|_{(m+\frac{1}{2})} - \widetilde{\mathcal{I}}_j|_{(m-\frac{1}{2})}}{\Delta x_j}, \quad \frac{\partial p \delta_{ij}}{\partial x_j} \simeq \frac{\widetilde{\mathcal{G}}_{ij}|_{(m+\frac{1}{2})} - \widetilde{\mathcal{G}}_{ij}|_{(m-\frac{1}{2})}}{\Delta x_j}, \quad \frac{\partial u_j p}{\partial x_j} \simeq \frac{\widetilde{\mathcal{P}}_j|_{(m+\frac{1}{2})} - \widetilde{\mathcal{P}}_j|_{(m-\frac{1}{2})}}{\Delta x_j}.$$

m is the computational node index, ρ is the density, u_i is the Cartesian velocity component in the ith-direction, p is the pressure, k is the kinetic energy, and e is the internal energy, respectively. Those numerical fluxes are derived to numerically replicate the three analytical relations that the governing equations (i.e., Euler equations) analytically hold: 1) the kinetic energy equation is derived from the mass and momentum equations, 2) the internal energy and kinetic energy are convected by the same convective velocity, and 3) the pressure diffusion term in the total energy equation is the sum of the pressure work term in the kinetic energy equation and the pressure dilatation term in the internal energy equation. Tamaki et al. [6] described the numerical meaning of the analytical relations in terms of discrete entropy conservation error.

Figure 1 shows the time evolution of the total kinetic energy and total entropy ($\Delta\rho s$=$\rho_0 s_0$-ρs, where s is the entropy) in an inviscid Taylor-Green vortex test conducted on a largely-distorted wavy computational grid. The initial Mach number is set at M_0=0.4. The second- and eighth-order KEEP schemes [3] conduct long-time stable computations without the aid of numerical dissipation, whereas the KEP schemes [4] computationally diverge. At the conference, we will comprehensively discuss the KEEP schemes, including the derivation background and the detailed numerical formulations of the KEEP schemes.

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