Anisotropic Mesh Modifications for the Moving Discontinuous Galerkin Method with Interface Condition Enforcement for Robust Simulations of High-Speed Viscous Flows

Eric J. Ching*, Andrew Kercher* and Andrew Corrigan*
Corresponding author: eric.ching@nrl.navy.mil

* Laboratories for Computational Physics and Fluid Dynamics,
U.S. Naval Research Laboratory, 4555 Overlook Ave SW, Washington, DC 20375.

The moving discontinuous Galerkin finite element method with interface condition enforcement (MDG-ICE) is an implicit shock fitting method capable of handling complex shock dynamics [1, 2]. The method is a unique variation of the well-known discontinuous Galerkin (DG) method. Specifically, neighboring elements are not coupled through interfacial, single-valued, numerical fluxes; instead, the conservation law and interface (Rankine-Hugoniot) conditions are directly discretized and the discrete domain geometry is treated as a variable. By simultaneously solving for the flow field and discrete geometry, MDG-ICE is able to produce extremely accurate solutions on very coarse grids as the grid points are automatically repositioned to fit shocks and resolve smooth regions of the flow with sharp gradients. This has significant advantages over traditional shock capturing approaches, such as artificial viscosity and limiting; the former relies on tunable parameters and introduces low-order errors into nominally high-order approximations, whereas the latter can obstruct iterative convergence and be difficult to apply to high-order solutions on curved elements with arbitrary shapes. Furthermore, since MDG-ICE adapts the grid to satisfy the the weak form, grid interfaces are automatically repositioned to fit a priori unknown shocks with arbitrary topology, thus overcoming key limitations of earlier explicit shock fitting methods.

Figure 1 presents results for supersonic viscous flow over a half-cylinder at Mach 5 and Reynolds number 10^4. A DG($\mathcal{P}1$) solution (where $\mathcal{P}_p$ denotes the space of polynomials of total degree p on simplicial elements) with artificial viscosity is compared with the MDG-ICE($\mathcal{P}4$) solution without any additional stabilization. The shock in the DG solution is noticeably smeared, and spurious oscillations are visible both at the shock and in the shock layer. On the other hand, the shock profile in the MDG-ICE solution is sharp and free from oscillations. MDG-ICE has also been demonstrated to achieve superior convergence rates and significantly higher accuracy than standard DG in boundary-layer problems.

Solution accuracy for approximations of external viscous flows over high-speed vehicles is highly sensitive to both grid alignment and grid resolution at the strong leading shock and within the boundary layer. We equip MDG-ICE with enhanced grid modifications in order to robustly compute high-order solutions on feature adapted grids from an initial coarse grid. In particular, we make use of the mesh-implied metric, which encodes information about the local element size and orientation (even on curved, anisotropic grids) [3]. We incorporate the metric into the

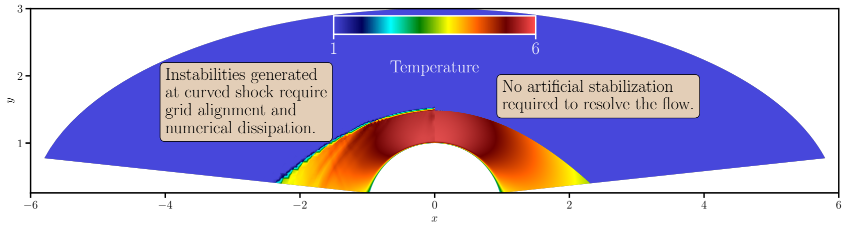

Figure 1: Comparison of the DG($\mathcal{P}1$) (with artificial viscosity) (left) and MDG-ICE($\mathcal{P}4$) (right) solutions to the viscous Mach 5 bow shock at $\mathrm{Re} = 10^4$ [2].

Levenberg–Marquardt nonlinear least-squares solver through an anisotropic grid regularization that inhibits grid motion in a manner that is inversely proportional to element lengths as defined by the metric. These tools are combined with the residual, a natural error indicator of least-squares methods, to drive anisotropic mesh refinement. This increases robustness and efficiency and overcomes the generation of "sliver" elements, as illustrated in Figure 2, caused by splitting edges of invalid elements that form as MDG-ICE adapts the grid to resolve the flow. These sliver elements introduce unnecessary degrees of freedom and may be undesirable for parametric studies in which, for example, the angle of attack is varied. Finally, we will apply these strategies to robustly and accurately compute multidimensional high-speed viscous flows with minimal degrees of freedom.

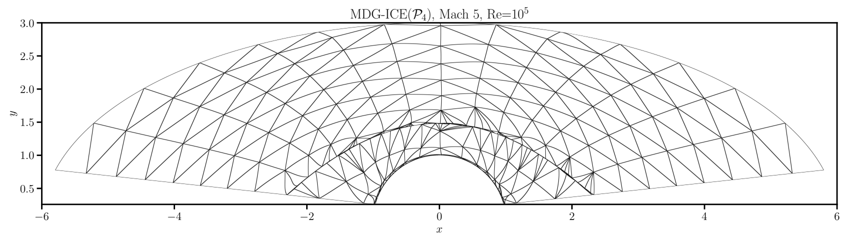

Figure 2: Final mesh for MDG-ICE solution to viscous Mach 5 bow shock at $\mathrm{Re} = 10^4$ [2].

References

[1] A. Corrigan, A.D. Kercher, and D.A. Kessler. A moving discontinuous Galerkin finite element method for flows with interfaces. *International Journal for Numerical Methods in Fluids*, 89(9):362–406, 2019.

[2] A.D. Kercher, A. Corrigan, and D.A. Kessler. The moving discontinuous Galerkin finite element method with interface condition enforcement for compressible viscous flows. *International Journal for Numerical Methods in Fluids*, 93(5):1490–1519, 2021.

[3] Krzysztof J Fidkowski. *A simplex cut-cell adaptive method for high-order discretizations of the compressible Navier-Stokes equations.* PhD thesis, Massachusetts Institute of Technology, 2007.