Output-Based Mesh Adaptation for High-Speed Flows

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Abstract: High-speed Computational Fluid Dynamics calculations often rely on structured meshes in order to facilitate aligning the mesh with shocks either manually or via some semi-automated process. The shock alignment of the mesh is often considered critical in order to obtain sufficiently accurate outputs such as integrated surface heat transfer. However, creating shock aligned structured meshes with complex geometries and/or multiple interacting shocks is challenging, and may not even be feasible for some configurations. Unstructured meshes offer greater flexibility focusing mesh resolution to capture complex geometry and flow features. This work demonstrates that integrated heat transfer computed using automated output-based adapted unstructured meshes can achieve higher degrees of accuracy compared shock fitted structured meshes for a set of canonical high-speed test cases.

1 Introduction

Properly resolving shocks for high-speed aerodynamic flow calculations is critical for accurate predictions of outputs of interest, such as lift, drag, or heat transfer to name a few. Structured meshes have historically been favored as the mesh can be reasonably aligned with shocks for relatively simple geometries without many shockwave interactions. However, generating structured meshes with appropriate shock resolution is challenging for more complex configurations. Furthermore, not all shock waves will have an impact on an output functional. A classical example of this is a "fish-tail" shock at high transonic speeds where the shocks are completely decoupled from the drag functional, and increasing resolution around the shock does not improve the predicted drag accuracy. Output-based mesh adaptation instead focuses mesh resolution on flow feature relevant to improving the accuracy in the output.

A series for high-speed flow examples are used here to demonstrate that output-based adapted unstructured meshes perform as well or better than shock fitted structured meshes.

2 Metric Optimization via Error Sampling and Synthesis

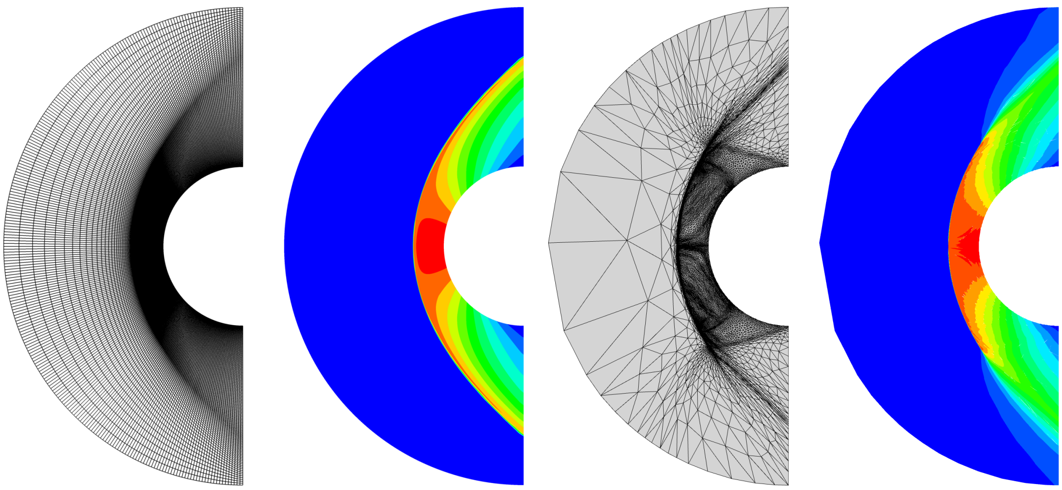
The Metric Optimization via Error Sampling and Synthesis (MOESS) [1, 2, 3, 4] mesh adaptation framework is used to generate optimal adapted meshes. These meshes are optimal in the sense that an output error indicator is minimized for a given cost. The Dual Weighted Residual (DWR) method [5] provides an error estimate for an output functional of interest, which is then localized to allow the adaptation process to target areas with high output error. Since directly optimizing the discrete mesh is intractable, the minimization is instead performed on a continuous metric field: a smooth field composed of symmetric positive definite matrices $\mathcal{M}(x)$. This approach is possible due to the duality between a discrete mesh, $\mathcal{T}_h$, and a continuous mesh represented by a Riemannian metric field $\mathcal{M} = \mathcal{M}(x)|_{x\in\Omega}$, as demonstrated by Loiselle [6, 7, 8].

This work uses Pointwise for isotropic mesh generation and the avro[9] software for metric-conforming mesh generation.

The MOESS algorithm is implemented using the Solution Adaptative Numerical Simulator (SANS) software where a Continuous Galerkin (CG) finite element method (FEM) stabilized by the Variational Multiscale Method with Discontinuous Subscales (VMSD)[10, 11] is used to discretize the Euler and Navier-Stokes equations.

3 Preliminary Results

An example comparison of the output-based adaption with radial, structured mesh shock fitting is shown for a 2D Mach 20 "Bova cylinder" [12] in Fig. 1. The structured-mesh shock fitting was determined heuristically based on static pressure rise, and the grid and density contours (calculated using OVERFLOW) are shown in Figs. 1a and 1b. The MOESS adapted grid and Mach contours, shown in Figs. 1c and 1d, successfully captures the shockwave along with the stagnation streamline as influencing the drag output functional. The stagnation streamline is completely missed in feature-based adaptation, while the output-based adaptation coarsens regions of the shock that do not influence the cylinder surface. Both qualities highlight the benefits of using output-based metrics.



(a) OVERFL: Grid (b) OVERFL: Density (c) MOESS: Grid (d) MOESS: Density

Figure 1: Bova cylinder [12] with $M_\infty = 20$, Inviscid

4 Conclusion and Future Work

The final conference paper will including additional high-speed test cases, along with a full order-of-accuracy analysis of the structured and unstructured adapted meshes.

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