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Dedicated to the Memory of Dr. Meng-Sing Liou (NASA Glenn Research Center:1987-2017)

On the Development of 'All Speed' Flux Formulae based on the AUSM Framework

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1 Abstract

This paper is written in honor and memory of my esteemed friend and colleague, Dr. Meng-Sing Liou, with whom I worked closely in the 1990s on the development of AUSM-family schemes and their extensions to reactive, 'all speed', and multiphase flows. The purpose of this paper is to revisit the thought processes and concepts that led to the rapid evolution of the AUSM family as a highly-efficient, highly accurate method for discretization of the Euler equations and their various extensions. No new results are presented; rather, the focus is on (re-) discovering the common threads that link the various members of the AUSM-family Special attention is given to the strategies that lead to accurate results at all flow speeds, and some reviews of more current work in this area are presented. The paper concludes with a few reflections, personal and otherwise, relating to my interactions with Dr. Liou over the years and to my view of the essential elements of AUSM-family schemes.
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2 A Brief History Lesson and Some Observations

This symposium honors the memory of Dr. Meng-Sing Liou, a preeminent researcher in CFD, a valued colleague and collaborator, and a close personal friend. I always called him 'Meng' and will refer to him as such for the remainder of this paper. In his landmark 1991 paper with Chris Steffen [1], Meng discovered a way to remove numerical diffusion inherent within flux-vector splitting (FVS) techniques without sacrificing stability. As most FVS schemes are O(n) and do not require complex matrix algebra, Meng's discovery paved the way for highly accurate, highly efficient, upwind-based discretization techniques extendable to reactive flows, real fluids, incompressible flows, and multi-phase flows, thus opening the door to a wide variety of applications. His initial scheme, termed the Advective Upwind Splitting Method (or AUSM), solved a long-standing problem – how does one remove numerical diffusion on a stationary contact wave while retaining the robustness and efficiency of a FVS? Meng's starting point was the van Leer FVS [2], first proposed in 1982 and later modified to preserve constancy of stagnation enthalpy by Hänel, et al. [3]. Van Leer himself recognized that his scheme would not capture a stationary contact wave without unphysical numerical diffusion – this property compromises the prediction of viscous flows, as the added numerical diffusion can be much greater than the physical diffusion supplied by the viscous stresses. Though higher-order extensions can partially alleviate the issue, it still remains. This is the primary reason why researchers worldwide gravitated to Roe's method in the 1980s when Navier-Stokes calculations became more routine. Meng's innovative concept started first with the splitting of the Euler flux into convective and pressure contributions, a process made easier by the Hänel simplification of van Leer's original scheme. He left the pressure flux contribution unchanged (at least initially) and focused his attention on reformulating the mass flux. His modification in retrospect is exceedingly simple: given that

$$F_c = \rho_L a_L M^+(M_L)\Phi_L + \rho_R a_R M^-(M_R)\Phi_R \tag{1}$$

according to van Leer, he replaced this with

$$F_c = \rho_L a_L \max(0, M^+(M_L) + M^-(M_R))\Phi_L + \rho_R a_R \min(0, M^+(M_L) + M^-(M_R))\Phi_R \tag{2}$$

thereby changing our field forever. It is easy enough to prove that this form will preserve a stationary contact wave – it is not so easy to show that there are additional diffusion mechanisms built in that will keep the scheme stable.

I first met Meng in ~1991 at a conference. I was introduced to him by my Ph. D. advisor, Dr. Scott McRae, and I requested a few of his papers, which were NASA TMs. I was trying to code up something that would preserve a boundary layer structure better than the van Leer methods that I was using. I ended up using a hybrid upwind / central difference method for my dissertation, as I wished to preserve a Jacobian matrix that had a 5-point stencil in the wall-normal directions. Later, as a post-doc at North Carolina A&T State University, I began to work on a more general code that adopted upwinding in all directions. At this time, I coded the original AUSM and found a way to construct a nearly exact linearization of it [4], as I was using (and still use) implicit schemes for time evolution

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I met Lou Povinelli (NASA Lewis Research Center, later Glenn Research Center) at some conference or workshop. Through him, I was able to obtain a summer appointment at LeRC in 1993 through the Institute for Computational Mechanics in Propulsion (ICOMP). ICOMP was set up to enable visiting researchers to interact with NASA personnel – some could work on-site (if granted access) but others were housed in the Ohio Aerospace Institute, a glass-walled building located just outside one of the gates. This was a very vibrant time for ICOMP, with the Center for the Modeling of Turbulence and Transition (CMOTT) being very active and with their being a number of permanent ICOMP employees housed at OAI. I had access to the base and frequently worked in Engineering Building 5, in a computer lab known as the 'Aquarium' that housed a number of SGI workstations, named after different types of fish. It was then that I first interacted directly with Meng, who had his office in this building. I don't recall whether Lou introduced us or whether I just introduced myself, but I would stop by frequently to discuss CFD-related matters. Meng was always kind and accommodating – a trait that remained with him for all the years that I knew him. At the time, I was working on multigrid methods for high-speed flows [5] and was doing a large number of hypersonic blunt-body simulations - the working code used AUSM. The properties of AUSM in resolving boundary-layers accurately and in avoiding the carbuncle response were very useful in this setting, but the inability of AUSM to capture non-grid aligned shocks monotonically was also a problem. I didn't know this at the time, but Meng was working on AUSM+, which was designed in part to remedy this issue. More about this later.

When working at OAI, I did not have an office – rather, I and several others worked at some stand-alone workstations that were arranged so that several people were grouped together. Two of my colleagues were Dr. Jaap van der Vegt, who was working as an ICOMP employee and is now a Professor of Mathematics at the University of Twente in the Netherlands, and Dr. Yasuhiro Wada, who was also a visiting researcher at ICOMP but was employed at the National Aerospace Laboratory in Japan. I became close friends with Jaap – I still remember him telling me the correct 'Dutch' way of pronouncing the famous painter Vincent van Gogh's last name…. He was working on Osher-type schemes, which I didn't quite understand at the time and still don't. He is a mathematician, pure and simple, and has made significant advances in discontinuous Galerkin methods in his work since ICOMP.

Yasuhiro Wada – where do I begin? I can't say that I knew him very well – there was a significant language barrier – but I did talk with him frequently while at OAI, and I remember his great sense of humor. Knowing of my issue in needing to eliminate AUSM-type shock oscillations, he gave me a preprint of the landmark paper that he wrote with Meng describing the AUSMDV schemes [6,7], and I spent much of my remaining time there trying to understand those schemes and working them into my code. Yasuhiro was not given base access, nor, for some reason, was he given access to some of the commonly-used NASA software, such as Tecplot. So he wrote his own graphics programs in the Postscript language! He would run his 1D code and you could watch the real-time evolution of various flow properties as well as a graphical picture of the various wave structures interacting with one another…. His amazing program included an exact Godunov scheme, Osher's scheme, Roe's scheme, and the various van Leer and AUSM-type schemes that he was working on. I am certain that Meng used a copy of this code for years afterward. A few of the great ideas from [6,7] are listed below.

- The introduction of a common (interface) speed of sound into the van Leer / AUSM family. There is no way to overstate the importance of this simple modification, as it enables Godunov-like solutions of various Riemann problems and led to many of the ideas that I will discuss subsequently. Here, one defines all left- and right-state Mach numbers using a common sound speed, calculated in some manner at the cell interface:
$$a_{1/2} = \frac{1}{2}(a_L + a_R), \quad \min(a_L, a_R), \quad \max(a_L, a_R), \ \sqrt{a_L a_R}, \ldots \tag{3}$$
- The introduction of 'pressure diffusion' into the mass flux – this is the AUSMDV modification in essence. This will be elaborated upon later, as it is critical to improving the performance of AUSM-type methods for non-grid-aligned discontinuities and some Riemann problems but leads to other complications, such as the appearance of the famous 'carbuncle' response.
- A multi-dimensional 'shock fix' that essentially eliminates the carbuncle response
- A sonic-point fix that removes the 'glitch' that most schemes have at sonic transitions.

Yasuhiro was a genius. Meng and I both knew this. He started with Meng's powerful AUSM idea and developed something completely original that moved the framework significantly forward. His untimely passing in 1995 due to a heart attack was a major loss to our field, but his ideas live on in later developments of the AUSM family. There is no telling what he would have accomplished had he lived.

Meng encouraged me to try Yasuhiro's scheme. At the same time, he was working on AUSM+ - I think that Yasuhiro knew about this, but I was in the dark. Meng was always somewhat secretive when he was building the next member of the AUSM family. He had received some criticism from the community when trying to publish the original AUSM scheme, mainly as it (as well as its 'parent', the famous van Leer (VL) FVS scheme) is somewhat *ad hoc*, lacking the mathematical framework of the Godunov-type methods. This may be one reason why researchers in the US were initially reluctant to adopt the AUSM-type methods, even as they became at least as accurate and always more efficient than the Godunov-based techniques. Meng was spending his time trying to prove several aspects of the performance of his new scheme – the AUSM+ paper [8] is filled with lemmas that serve to anchor various traits of the scheme. Meng's major problem with AUSMDV was the fact that it did naturally admit a carbuncle response. We knew that it was a consequence of the addition of 'pressure diffusion' in the mass flux – schemes that did not possess such (e.g. AUSM, VL) did not respond in this way, while schemes that did (unmodified AUSMDV, Roe) did. AUSM+ was not going to have this kind of diffusion mechanism, as far as Meng was concerned. He maintained this viewpoint for many years. I never coded AUSM+ during my initial stay at LeRC – the NASA TM that described it did not appear until 1994.

My work at NC A&T concluded in December of 1993, at which point I went to work as a contractor at NASA Langley before joining the faculty of NCSU in August of 1994. I can't recall what motivated me to create my own member of the AUSM-family, termed the 'Low-Diffusion Flux-Splitting Scheme' (LDFSS), but its genesis was another paper co-authored by Meng [9], this time with

Frederic Coquel of ONERA who also was a visiting scholar at ICOMP. This intensely mathematical paper finally concludes with a means of subtracting numerical diffusion inherent within the van Leer (Hänel) scheme via the introduction of an intermediate state determined from the Osher-Solomon approximate Riemann solution. This enables capturing of a stationary contact wave in a manner different from the advective upwinding approach used in AUSM, AUSM+, and AUSMDV. The first version of LDFSS was a modification of Coquel-Liou that avoids the calculation of the Osher state - this scheme captures normal shocks in a manner similar to van Leer (Hänel), removes dissipation at a stationary contact wave (and thus is accurate within a boundary layer), but also can exhibit oscillations for non-grid aligned oblique shocks, similar to AUSM and AUSM+. A solution to the latter problem was found by incorporating a pressure-diffusion mechanism similar to that proposed by Wada and Liou, but differing in implementation. This approach would admit, as expected, a carbuncle response, and when the 1995 conference paper [10] (incidentally presented in the same session as the first AIAA presentation of AUSM+) was submitted for archival publication, a not-so-kind reviewer pointed this out and recommended rejection. The reviewer was correct – the initial LDFSS was no better than AUSMDV, Roe, and others in this regard. This spurred one of the best guesses of my life – I was able to discover an alternative pressure-diffusion mechanism that could suppress the carbuncle instability without violating the constraints of a stationary contact discontinuity. [11] From this point, LDFSS became my group's workhorse tool and was implemented into several production-level codes within NASA, Dow Chemical, and other places. Its most prominent use (outside of our own work) is as the primary method of discretization for NASA's VULCAN code, a widely-used tool for propulsion-system analysis. Meng knew of the LDFSS development, reviewed the pre-prints and was certainly supportive and encouraging, but again, he did not believe in the need for pressure diffusion at the time.

I returned to NASA Glenn in the summer of 1996, again stationed at ICOMP in OAI. I had discovered a need for time-derivative preconditioning as a means of extending our solver's capabilities and had begun studying the problem within the context of LDFSS. Meng was also interested in this issue but had not yet focused his attention fully on it. The fact that the preconditioned system admits a different set of eigenvalues naturally led to the conclusion that one could base the interface Mach number calculation on a 'sound speed' determined from the pseudo-acoustic speeds – this eventually led to the concept of a 'numerical speed of sound' [12] and a NASA Glenn Technology Award for this concept, shared by Meng and me. A representative form for this numerical speed of sound is as follows:

$$\tilde{a}_{1/2} = a_{1/2}\frac{\sqrt{(1-M_{ref}^2)M_{1/2}^2 + 4M_{ref}^2}}{(1+M_{ref}^2)}, M_{ref}^2 = V_{ref}^2 / a_{1/2}^2 = \min(|\vec{V}|^2, V_\infty{}^2, a_{1/2}^2) / a_{1/2}^2 \qquad (4)$$

In common with most methods that use time-derivative preconditioning, there is a cutoff velocity that must be prescribed – this choice will be case-dependent. The problem is that this simple fix alone does not work. There are two issues to address. The first is that the pressure diffusion mechanism (inherent in AUSMDV and LDFSS but absent in AUSM and AUSM+) needs to be scaled upward as the local reference Mach number decreases to preserve pressure-velocity coupling. This need was later quantified through perturbation analysis by Meng in his AUSM+-up paper [13] but at the time, it was more heuristic in nature, motivated by the presence of similar terms in the Rhie-Chow collocated mesh method and by Lax-Friedrichs or Roe-type flux constructions based on the preconditioned equation system. Since pressure diffusion was absent in AUSM+, it had to be added; procedures for doing such were first described in [14]. It should be mentioned that the LDFSS extension is absent from this publication. At the time, I was funded by Meng via the ICOMP arrangement and by a later follow-on grant to NCSU, and as such, I felt it important to concentrate my work on AUSM+ and AUSMDV, even though all techniques developed were first tested using LDFSS.

The second issue had, to this point, completely escaped notice within the small community that was working with the AUSM family of schemes. Every technique developed to that point had retained the classic van Leer (1982) pressure splitting for the most part. Meng's innovative concept of separately discretizing the convective and pressure parts of the interface flux had led to the pressure component being essentially ignored, with the only significant modification being his use of higher-degree polynomials in Mach number for his AUSM+ scheme. I discovered that this term could become incredibly diffusive at very low Mach numbers, regardless of whether the term was formulated using the physical sound speed or the numerical one. While at NASA, I experimented with different ways of formulating the left- and right-state Mach numbers to reduce the diffusion – one solution was found by the following:

$$
\begin{aligned}
\tilde{M}_L &= \frac{1}{2}(1+M_{ref}^2)M_L + \frac{1}{2}(1-M_{ref}^2)M_R \\
\tilde{M}_R &= \frac{1}{2}(1+M_{ref}^2)M_R + \frac{1}{2}(1-M_{ref}^2)M_L
\end{aligned}
\tag{5}
$$

This simple modification, along with the scaling of the pressure diffusion term by the inverse square of the reference Mach number, enabled accurate solutions to be obtained for gas-phase flows at all speeds. The 'numerical sound speed' concept [12] codified these ideas, setting up a systematic approach by which any AUSM-type scheme for gas dynamics could be extended to operate at all speeds.

The next phase in my collaboration with Meng focused on the extension of AUSM-type schemes to real fluids with phase transitions. This work took place from 1998 to 2000 and was the only part of our collaboration that was actually funded by a grant to NCSU. My motivation at the time was toward predictions of supercritical fluids with application to coating processes and hydrocarbon fuel injection; Meng's interest was spurred by the need to predict cavitation within some of the turbopumps used in NASA's Space Shuttle main engines as well as other liquid transport / storage issues associated with the Space Shuttle. Our starting point was a homogeneous equilibrium two-phase flow model constructed by embedding results from a vapor-liquid equilibrium analysis into a real-fluid (single component) thermodynamic description provided by a generalized equation of state. We used the Peng-Robinson and Sanchez-Lacombe state equations. An advantage of the homogeneous equilibrium model is that it leads to a hyperbolic equation system. Time-derivative preconditioning methods, specialized for the density-based formulation that the homogeneous equilibrium model requires (in the two-phase region, pressure is solely a function of temperature), can be applied. The essential modifications necessary to extend the 'all speed' flux formulae developed earlier to function for this system involved the replacement of scaling terms involving the pressure with the combination of density times sound speed squared. In several places within the gas-dynamic AUSM framework, the assumption that $p/\rho a^2$ is of order unity is made implicitly – this statement is not true for a purely incompressible flow, where the absolute pressure is arbitrary, nor for a general real fluid.

The procedures outlined in [15] and applied to AUSM+(P) ('P' for preconditioned), while effective, were somewhat cumbersome to implement, particularly for the pressure splitting, where a linearized treatment was employed to separate the two diffusion mechanisms present – a pressure diffusion term proportional to the pressure difference across an interface, and a 'velocity diffusion' term, the scaling of which is the root cause of over-diffusive behavior at low Mach number. A better approach was outlined in [16], in which the LDFSS framework was re-worked into an 'all speed', real-fluid flux formulation that was also valid in the incompressible limit of the sound speed approaching infinity. One important outcome of this paper was a re-formulation of the pressure flux splitting into its separate components:

$$
\begin{aligned}
p_{1/2} &= P^{+}(M_L)p_L + P^{+}(M_R)p_R \\
&= \frac{1}{2}(p_L + p_R) \quad \text{[average pressure]} \\
&+ \frac{1}{2}(P^{+}(M_L) - P^{+}(M_R))(p_L - p_R) \quad \text{[pressure diffusion]} \\
&+ \frac{1}{2}(p_L + p_R)(P^{+}(M_L) + P^{+}(M_R) - 1) \quad \text{[velocity diffusion]}
\end{aligned} \tag{6}
$$

It is not obvious at first glance that the last term represents 'velocity diffusion', but it is easily seen if one substitutes the simplest subsonic pressure splitting $P^{\pm}(M) = 1/2(1 \pm M)$ into the above. The velocity diffusion term becomes

$$
\frac{1}{4}(p_L + p_R)(M_L - M_R) = \frac{1}{4}\frac{(p_L + p_R)}{a_{1/2}}(u_L - u_R) \tag{7}
$$

It is here that the need to replace the average pressure by $\rho_{1/2}a_{1/2}^2$ becomes more apparent, as does the need to scale this term by the square of the reference Mach number to remove the dependence on the sound speed in the limit of an incompressible flow. The final form becomes

$$
\begin{aligned}
p_{1/2} &= P^{+}(M_L)p_L + P^{+}(M_R)p_R \\
&= \frac{1}{2}(p_L + p_R) \quad \text{[average pressure]} \\
&+ \frac{1}{2}(P^{+}(M_L) - P^{+}(M_R))(p_L - p_R) \quad \text{[pressure diffusion]} \\
&+ \frac{1}{2}\rho_{1/2}V_{ref,1/2}^2(P^{+}(M_L) + P^{+}(M_R) - 1) \quad \text{[velocity diffusion]}
\end{aligned} \tag{8}
$$

To leading order in Mach number, the velocity diffusion term becomes

$$
\frac{1}{2}\rho_{1/2}V_{ref,1/2}^2 \frac{(1 + M_{ref,1/2}^2)}{\sqrt{(1 - M_{ref,1/2}^2)u_{1/2}^2 + 4V_{ref,1/2}^2}}(u_L - u_R), \tag{9}
$$

which scales as the reference velocity for low Mach numbers. This form, along with modifications made to the mass-flux pressure splitting to enable real-fluid / 'all-speed' functionality (discussed later), were the key components of LDFSS-2001, the final form of development of the LDFSS branch of the AUSM family. [16] This paper was presented as an invited talk at the 2001 AIAA Computational Fluid Dynamics conference. One of my regrets is that I never submitted this paper for archival publication, as it contains, in my view, the most complete exposition of the concepts described in brief in this paper. LDFSS-2001 was later applied to incompressible and compressible two-phase flows described by a homogeneous mixture model with interface sharpening [17, 18] and has been the cornerstone of our subsequent work. One observation made in [16] was that the choice of scaling parameter for ensuring low-Mach functionality was somewhat arbitrary – either the reference velocity or the numerical sound speed or other combinations could be utilized. Later developments of the AUSM family have leveraged this flexibility.

By this point, my formal collaboration with Meng had ended, though we kept in touch by phone, e-mail, and meetings at conferences. I was working with others at NASA Glenn from ~2002-2005 on rocket-based combined cycle engine concepts and would often visit him during my trips there. We would always manage find a place with Indian buffet food for lunch. Meng moved forward with his branch of the AUSM family, leveraging some of the ideas that we developed in the construction of AUSM+-up, which was his last published version. [13] As mentioned, he was very reluctant to compromise AUSM's natural insensitivity to the carbuncle response, but in AUSM+-up, a pressure diffusion mechanism finally appeared, along with an augmentation of the velocity-diffusion term present in the pressure splitting:

$$p_{1/2} = P^{+}_{(5)}(M_L)p_L + P^{-}_{(5)}(M_R)p_R + K_u P^{+}_{(5)}P^{-}_{(5)}\rho_{1/2}(f_a a_{1/2})(u_L - u_R), \quad K_u = 3/2 \tag{10}$$

Here, the 5th order polynomials first proposed for use in AUSM+ were employed with a modification designed to ensure that the velocity diffusion term inherent in the original part of the splitting also scaled properly with Mach number. For subsonic flows, one can express the 5th order polynomials as follows:

$$P^{\pm}_{(5)}(M) = \frac{1}{2}(1 \pm e(M)); \, e(M) = \left(\frac{3}{2} + 2\alpha\right)M - \left(\frac{1}{2} + 4\alpha\right)M^3 + 2\alpha M^5; \quad \alpha = \frac{3}{16}(5f_a^2 - 4) \tag{11}$$

Substituting for α gives

$$e(M) = \frac{15}{8}f_a^2 M - \frac{15}{4}\left(f_a^2 - \frac{2}{3}\right)M^3 + \frac{15}{8}(f_a^2 - \frac{4}{5})M^5 \tag{12}$$

Implementing this using the decomposition of Eq. (6) and restricting to leading order in Mach number gives

$$\begin{aligned}
&p_{1/2} \approx \frac{1}{2}(p_L + p_R) \quad \text{[average pressure]}\\
&+\frac{1}{2}[\frac{15}{16}f_a^2(M_L + M_R)](p_L - p_R) \quad \text{[pressure diffusion]}\\
&+\frac{1}{2}(p_L + p_R)[\frac{15}{16}f_a^2(M_L - M_R))] \quad \text{[velocity diffusion]}\\
&+\mathrm{K}_u\rho_{1/2}(f_a a_{1/2})\frac{1}{4}[1 + \frac{15}{8}f_a^2(M_L - M_R)](u_L - u_R) \quad \text{[additional velocity diffusion]}
\end{aligned} \tag{13}$$

Meng's AUSM+-up form does not substitute for the average pressure in the third term. If this is done, and the Mach number definitions employed in [13] utilized, then one obtains

$$\begin{aligned}
&p_{1/2} \approx = \frac{1}{2}(p_L + p_R) \quad \text{[average pressure]}\\
&+\frac{1}{2}[\frac{15}{16}f_a^2(M_L + M_R)](p_L - p_R) \quad \text{[pressure diffusion]}\\
&+\rho_{1/2}f_a^2 a_{1/2}\frac{15}{16}(u_L - u_R) \quad \text{[velocity diffusion]}\\
&+\mathrm{K}_u\rho_{1/2}(f_a a_{1/2})\frac{1}{4}[1 + \frac{15}{8}f_a^2(M_L - M_R)](u_L - u_R) \quad \text{[additional velocity diffusion]}
\end{aligned} \tag{14}$$

The scaling function f_a is $M_o(2 - M_o)$, where $M_o^2 = \min(1, \max(M_{1/2}^2, M_\infty^2))$, meaning that the additional velocity diffusion term scales similarly to that used in LDFSS as the Mach number approaches low values, but that the 'natural' velocity diffusion term reduces more quickly. f_a is a direction-dependent quantity, as is the scaling factor that connects $\tilde{a}_{1/2}$ to $a_{1/2}$ in Eq. (4). If f_a is set to unity, thus removing the low-Mach modifications, both terms scale as the physical speed of sound and are both important.

Looking at the evolution of the low-Mach modifications from a distance (as I am doing now), it is clear that Meng, while recognizing (and later proving) the correctness of the scaling arguments that we developed, wished to remove the direct connection with the eigensystem of the preconditioned equations as much as possible. LDFSS has retained this connection. The structure of the leading diffusion terms is the same for both strategies, but the multiplicative constants are different, leading to different balances between advective upwinding, pressure diffusion, and velocity diffusion. One constraint imposed on the LDFSS development is the notion of an incompressible-limiting form, defined as the physical sound speed approaching infinity and the pressure level becoming arbitrary. Application of this constraint should lead to a similar structure for numerical diffusion, with no terms vanishing and no terms approaching infinity as the Mach number approaches zero. The application of this principle led to LDFSS-2001; parts of it have found their way into other developments of the AUSM-family.

The remainder of this paper focuses on a comparative analysis of some of the numerical diffusion mechanisms found within the AUSM-family members. Included in this comparison are the AUSMPW+ scheme ('PW' for 'pressure weighting'), developed by Prof. Kim and his students [19], and the SLAU2 ('SLAU' for Simple Low-dissipation Advective Upstream) scheme, developed by Kitamura, Shima, and co-workers [20,21]. In terms of a chronological history, AUSMPW+ was developed slightly later than LDFSS and was focused strongly toward re-entry predictions; SLAU and its successor SLAU2 were developed later still, from 2009 onward. While it is certain that other AUSM-type schemes have been and continue to be developed, many are essentially re-inventions of the wheel; AUSMPW+ and SLAU added some new elements to the canon and deserve further study.

2 AUSM-Family Interface Pressure: A Comparative Analysis

Differences among the AUSM family with respect to the interface pressure treatment can be substantial. While most variants adopt the van Leer / Liou pressure splitting forms (1st degree, 3rd degree and 5th degree polynomials), they differ in the specific choices employed and also in the scaling (if applied) of the various terms. As discussed above, the decomposition of the interface pressure into a cell average pressure, a pressure diffusion term, and a velocity diffusion term serves to highlight similarities and differences among the various approaches. Here, we list a few forms in chronological order of appearance in the literature

Van Leer, AUSM, AUSM+, AUSMDV, AUSMPW, LDFSS-1997 interface pressure:

$$\begin{aligned} p_{1/2} &= \frac{1}{2}(p_L + p_R) + \frac{1}{2}(P^+(M_L) - P^+(M_R))(p_L - p_R) \\ &+ \frac{1}{2}(p_L + p_R)(P^+(M_L) + P^+(M_R) - 1) \end{aligned} \tag{15}$$

LDFSS-2001 interface pressure:

$$\begin{aligned} p_{1/2} &= \frac{1}{2}(p_L + p_R) + \frac{1}{2}(P^+(M_L) - P^+(M_R))(p_L - p_R) \\ &+ \rho_{1/2} a_{1/2}^2 (P^+(M_L) + P^+(M_R) - 1), \end{aligned} \tag{16}$$

AUSM+-up interface pressure

$$\begin{aligned} p_{1/2} &= \frac{1}{2}(p_L + p_R) + \frac{1}{2}(P^+(M_L) - P^+(M_R))(p_L - p_R) \\ &+ \frac{1}{2}(p_L + p_R)(P^+(M_L) + P^+(M_R) - 1) \\ &+ K_u P^+(M_L) P^+(M_R) \rho_{1/2} a_{1/2} (u_L - u_R) \end{aligned} \tag{17}$$

SLAU2 interface pressure:

$$\begin{aligned} p_{1/2} &= \frac{1}{2}(p_L + p_R) + \frac{1}{2}(P^+(M_L) - P^+(M_R))(p_L - p_R) \\ &+ \rho_{1/2} a_{1/2} \sqrt{\tfrac{1}{2}(\vec{V}_L \cdot \vec{V}_L + \vec{V}_R \cdot \vec{V}_R)}(P^+(M_L) + P^+(M_R) - 1), \end{aligned} \tag{18}$$

The scaling of the velocity diffusion term is clearly different among all methods. Specializing to perfect-gas subsonic flow and adopting the simplest pressure splitting to focus on the leading terms, we have

$$
\begin{aligned}
&\frac{1}{2}\rho_{1/2}a_{1/2}[\frac{1}{\gamma}](u_L - u_R) \quad \text{(VL, AUSM, AUSM+, AUSMDV, AUSMPW, LDFSS-1997)} \\
&\frac{1}{2}\rho_{1/2}a_{1/2}(u_L - u_R) \quad \text{(LDFSS-2001)} \\
&\frac{1}{2}\rho_{1/2}a_{1/2}[\frac{1}{\gamma} + K_u\frac{1}{4}(1+M_L)(1-M_R)](u_L - u_R) \quad \text{(AUSM+-up)} \\
&\frac{1}{2}\rho_{1/2}a_{1/2}\frac{\sqrt{\frac{1}{2}(\vec{V}_L\cdot\vec{V}_L + \vec{V}_R\cdot\vec{V}_R)}}{a_{1/2}}(u_L - u_R) \quad \text{(SLAU2)}
\end{aligned}
\tag{19}
$$

A few conclusions can be drawn. The scaling of the velocity diffusion term for both SLAU2 and LDFSS is not dependent on interface orientation, but the difference between the levels of diffusion provided could be substantial, depending on the local Mach number and the orientation of the flow direction with respect to an interface normal. The scaling term is more similar between LDFSS and AUSM+-up, but there is a direction-dependent component to the latter. Further, the retention of the average pressure as a scaling quantity for the velocity diffusion term (as done in the earlier models) leads to an additional factor of $1/\gamma$ for a perfect gas. Low-speed modifications would require that the scaling of the velocity term by the sound speed be replaced by something proportional to the velocity magnitude – the SLAU2 form does this for all speeds, while modifications discussed earlier for LDFSS and AUSM+-up would apply this correction only for subsonic flows, with its effect diminishing as a sonic state is reached. The SLAU2 form is also local, and the velocity diffusion term could vanish in a stagnation region; the low-speed forms for LDFSS and AUSM+-up would be limited by some cutoff velocity. The pressure diffusion parts of the models are the same unless low Mach number approximations are applied.

If the incompressible limit is applied ($a_{1/2} \to \infty$) and gauge pressures are utilized ($p \to p' = p - p_{ref}$), then the following reduced forms are obtained:

$$
\begin{aligned}
&p_{1/2} = \frac{1}{2}(p'_L + p'_R) \quad \text{(VL, AUSM+, AUSMDV, AUSMPW, LDFSS-1997)} \\
&p_{1/2} = \frac{1}{2}(p'_L + p'_R) + \frac{1}{\sqrt{u_{1/2}^2 + 4V_{ref,1/2}^2}}[\frac{1}{4}(u_L + u_R)(p'_L - p'_R) + \frac{1}{2}\rho_{1/2}V_{ref,1/2}^2(u_L - u_R)] \quad \text{(LDFSS-2001)} \\
&p_{1/2} = \frac{1}{2}(p'_L + p'_R) + \frac{K_u}{4}\rho_{1/2}V_{ref,1/2}(u_L - u_R) \quad \text{(AUSM+-up)} \\
&p_{1/2} = \frac{1}{2}(p'_L + p'_R) + \frac{1}{2}\rho_{1/2}\sqrt{\frac{1}{2}(\vec{V}_L\cdot\vec{V}_L + \vec{V}_R\cdot\vec{V}_R)}(u_L - u_R) \quad \text{(SLAU2)}
\end{aligned}
\tag{20}
$$

It is interesting that the use of gauge pressures eliminates all sources of numerical diffusion in the incompressible limit for the <u>original</u> interface pressure model! This does not mean very much– the pressure level itself is arbitrary, but not necessarily zero, in an incompressible flow. The scheme should not care which pressure level one selects as only changes in pressure are important. Diffusive mechanisms are retained for the later developments, with LDFSS-2001 (by construction) keeping both pressure diffusion and velocity diffusion in the incompressible limit. Pressure diffusion is removed in the AUSM+-up and SLAU2 forms, though velocity diffusion is retained and is similar to that of LDFSS-2001.

3 AUSM-Family Mass Flux: A Comparative Analysis of Pressure Diffusion Mechanisms

The original AUSM scheme upwinds a vector of advected quantities ($[\rho, \rho\vec{V}, \rho h_0]^T$) based on the sign of an interface velocity; AUSMDV and some later developments upwind a different set of advected variables ($[1, \vec{V}, h_0]^T$) based on the sign of the interface mass flux. LDFSS does neither, adopting a form very similar to the original van Leer scheme. These distinctions may be important in some instances, but the essential behavior of these schemes relates more to the mass flux itself, and it is here that we focus our attention. Generally speaking, the mass flux for an AUSM-family member can be written as

$$\dot{m} = \frac{1}{2}(\rho_L u_L + \rho_R u_R) + \frac{1}{2}|u_{1/2}|(\rho_L - \rho_R) + \dot{m}_v + \dot{m}_p \tag{21}$$

where the first part is an average mass flux, the second part is density-diffusion term that arises from advective upwinding, and the third and fourth parts are velocity and pressure diffusion mechanisms. It should be recognized that these distinctions do not arise explicitly for most AUSM-family members – they are embedded in the behavior of the van Leer / Liou Mach number polynomials and other functional forms. Only SLAU explicitly employs a construction similar to this. Precise expressions for the average velocity $|u_{1/2}|$ differ only slightly among the schemes away from sonic points. Near sonic points, the velocity diffusion mechanism can become important – this form can generally be expressed as $\dot{m}_v \sim \rho_{1/2} M_{1/2}(u_L - u_R)$ but precise forms again are somewhat scheme-dependent. The construction of the pressure-diffusion term constitutes the major difference among members of the AUSM-family. Again, precise forms for this term can be difficult to extract from the published flux formulations, but it is sufficient to observe the behavior of $\dot{m}$ as the directional velocity components u_L, u_R vanish. The remaining contribution to the mass flux will be the pressure diffusion term, which should be proportional to a pressure difference across a cell interface for the scheme to capture a stationary contact wave exactly. Limiting forms for the pressure diffusion term are shown below for a chronological listing of AUSM-family schemes.

$$\begin{aligned}
&\dot{m}_p = 0 \quad \text{[AUSM, AUSM+; 1991-1995]} \\
&\dot{m}_p = K_p \frac{a_{1/2}}{\frac{p_L}{\rho_L} + \frac{p_R}{\rho_R}} (p_L - p_R) \quad \text{[AUSMDV; 1994-1995]} \\
&\dot{m}_p = K_p a_{1/2} \left[\left(1 - \delta \frac{|p_L - p_R|\, p_{1/2}}{p_L p_R}\right) \frac{\rho_{1/2}}{p_{1/2}} (p_L - p_R) + \delta \frac{|p_L - p_R|\, p_{1/2}}{p_L p_R} (\rho_L - \rho_R) \right] \quad \text{[LDFSS;1995-1997]} \\
&\dot{m}_p = K_p a_{1/2} \frac{\rho_{L,R}}{p_{1/2}} (p_L - p_R) \quad \text{[AUSMPW+; 2001]} \\
&\dot{m}_p = K_p \frac{\tilde{a}_{1/2}}{V^2_{ref,1/2}} (p_L - p_R) \quad \text{[LDFSS-2001]} \\
&\dot{m}_p = K_p \frac{1}{f_a a_{1/2}} \frac{\rho_{L,R}}{\rho_{1/2}} (p_L - p_R) \quad \text{[AUSM+-up; 2006]} \\
&\dot{m}_p = K_p \frac{1}{a_{1/2}} \left(1 - \min(1.0, \frac{\sqrt{\frac{1}{2}(\vec{V}_L \cdot \vec{V}_L + \vec{V}_R \cdot \vec{V}_R)}}{a_{1/2}})\right)^2 (p_L - p_R) \quad \text{[SLAU2; 2009-2012]}
\end{aligned} \tag{22}$$

Here (and before), the '1/2' subscript denotes evaluation at an arithmetically-averaged state, while the 'L/R' notation denotes evaluation at either the left or the right state, depending on the sign of the interface Mach number or mass flux. Only AUSM and AUSM+ possess no pressure diffusion term – its utility in suppressing non-monotone behavior in boundary layers and at non-grid aligned shock waves eventually led to its inclusion in the later developments. Of the schemes listed above, LDFSS-1997 possesses the most complex form, as it contains a built-in mechanism to suppress the carbuncle instability. Most of the later developments also contain a suppression mechanism of one form or another. Only LDFSS-2001 and AUSM+-up possess the correct scaling in the incompressible limit (inversely with a reference velocity) – the pressure diffusion vanishes for the other models in this limit, providing no means of pressure-velocity coupling. The inclusion of the pressure as a direct scaling factor in AUSMDV, LDFSS-1997, and AUSMPW+ makes these schemes unsuitable as written for incompressible flows, in which pressure can be set arbitrarily. SLAU2 uses the most multi-dimensional velocity information; the others (excepting the low Mach number terms) use directional velocity information only in establishing the interface flux.

4 Conclusions and Some Personal Reflections

In this paper, I have attempted to provide a brief history of the early years of AUSM-family development, focusing most specifically on the work that Meng and I performed in the 1990s but trying also to tie those developments into earlier and later activities and to give the reader an idea of the thought processes at the time. I think that it is fair to state that the ICOMP visiting scholars program (Wada, Coquel, myself, others later) as facilitated by Lou Povinelli and Meng provided a route for the rapid advancement of the initial AUSM idea. He encouraged a diversity of opinion and never once attempted to constrain our ideas to whatever direction he himself was pursuing. Meng's vision of a simple, accurate Riemann solver has been proven correct. The literature abounds with successful applications of AUSM family schemes for all classes of fluid-dynamic problems, and the AUSM family has become part of the state of the practice in production-level CFD.

I would leave the reader with a few guiding principles that have advanced AUSM-family development throughout the years. The first is the need to capture exactly a stationary contact wave so that boundary layers can be captured accurately. The second is to maintain O(n) complexity so that extensions to multi-component systems and some multi-phase systems become trivial. The third is the need to introduce an amount of 'pressure diffusion' into the mass flux to remove unwanted oscillations and to control this addition so that the carbuncle response is at least suppressed, if not eliminated entirely. The fourth is the need to avoid the use of the pressure as a direct scaling factor so that extensions to incompressible, real-fluid, and multi-phase flows become possible. The fifth is to ensure that the pressure-diffusion term in the mass flux scales inversely and that the velocity-diffusion term in the interface pressure scales directly with a reference Mach number as the Mach number is lowered so that accurate solutions can be obtained at all speeds. Research issues necessary to advance the family hinge, I believe, on the proper introduction of multi-dimensional information and on the construction of a single scheme that seamlessly performs well at all speeds. Right now, one would not want to run an 'all speed' formulation for a 1D shock-tube problem or for a hypersonic bluff-body problem just as one would not want to apply a standard formulation for an incompressible driven-cavity problem. The need to select a reference velocity for low-speed applications also is a limiting factor for some methods. It also must be stated that the use of higher-order reconstruction methods renders differences between members of the AUSM family less noticeable – the low-diffusion characteristics that are so important for 1st or 2nd order predictions become less meaningful when higher-order techniques such as discontinuous Galerkin or flux reconstruction are employed.

In closing, I will provide a few thoughts on my interactions with Meng since we completed our work in ~2000. We kept in touch – he would call me regularly to see if I had any students that he could hire as post-docs and just to catch up. He always cared about my career – he understood that to keep my job, I'd have to move away from our areas of mutual interest. He had to do the same as NASA's focus areas changed. We would also meet during my periodic visits to NASA Glenn and at various conferences, always seeking some cheap Indian food if we could find it. He wrote a few recommendation letters for me – one for my initial appointment at NCSU, another for my tenure, another one for my (unsuccessful) application as AIAA Fellow, and a couple of others that I won't discuss. I wrote one for him for his (successful) AIAA Fellow application and also participated in a review of NASA Glenn's work in CFD by his invitation. Meng was not a self-promotor, and the importance of his work (at least in the U.S.) was not as recognized as it should have been, but he was a tireless innovator and opened the doors to even more innovation by virtue of his kind, patient manner with younger researchers such as myself. He ended his career as a Senior Technologist at NASA – the highest non-managerial level in that organization. That, plus the thousands of citations to his work, says enough. I had not seen him much in recent years and had been thinking about him when I heard from Prof. Yang-Yao Niu that his situation had turned for the worst. I was able to communicate with him one last time and was able to thank him for his guidance and friendship. His legacy lives on, and I miss him greatly.

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