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Refinement of Design Variables for Aerodynamic Shape Optimization

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Abstract: To investigate the impact of distribution of design variables on global optimization design, a determination strategy of variables and corresponding design space adjustment method based on adjoint surface sensitivities are introduced and applied in a two-dimensional inviscid airfoil optimization problem. The refinement strategy results in solutions with lower drag compared with cases of uniform design variables, and it may possess capacity to resist the impact of inferior observational data. The best geometry in this work is obtained with 11 nonuniform design variables of which the drag coefficient is 358.50 counts, 19.73 counts lower than that of uniform arrangement. Experiments show that there is strong correlation between effectivity of the refinement strategy and dimensionality which will be further investigated.

Keywords: surface sensitivities, design variables, design space, Free-form deformation, aerodynamic wave drag.

1 Introduction

With consistent advancements of computational fluid dynamics (CFD) and computer hardware, aerodynamic shape optimization (ASO) has matured to a stage that it has been successfully incorporated into the aircraft design. Improving a current solution commonly uses one of two approaches: gradient-based or global search algorithms. Both of the approaches are applied in the field of aerodynamic shape optimization, although gradient-based approaches are more popular. Gradient-based methods usually use the local gradients to dictate the next direction of searching, with the forms of steepest descent, Newton's method, conjugate gradient methods, and quasi-Newton methods. Despite tending to reach the local minima, gradient-based approaches have found use within aerodynamic shape optimization community, for example the work of Bisson [1] who used sequential-quadratic programming (SQP) method for drag minimization of three benchmark problems, the work performed by Onera [2] in which optimization convergence and the ability of adjoint-based optimizations were investigated on two test cases with the in-house Onera *elsA* software, and the work of Copeland et al [3] who developed a continuous formulation of the adjoint equations for thermochemical non-equilibrium optimization.

Gradient-based optimization algorithms are more popular compared to global search algorithms primarily because of the requirement to minimize the number of objective function evaluations, which is CFD simulations in ASO. Despite of efficiency advantages gradient-based approaches may not work well with discrete variables, discontinuity in objective function or multimodality in design space. In these cases, global search approaches have higher possibility to reach the overall global optimum. Global search approaches avoid computing and relying on the gradient, instead often mimic swarm behaviors and natural processes such as genetic algorithms (GA) [4], particle swarm optimization [5], simulated annealing (SA) [6], gravitational search algorithm (GSA) [7], and so on. Tremendous researches on evolutionary searching methods have been made to tackle aerodynamic optimization problems. Main advantages of such methods are portability and the possibility to explore design space thoroughly. However, the costs are quite large, and even prohibitive because high-fidelity aerodynamic performance analyses are now necessary and many simulations of the objective function are needed, especially when dimensionality increases.

The issue of impacts of dimensionality on the optimization process has been investigated. Some data [8, 9] shows that as dimension of design space increases, the optimum design monotonically improves, and appears to asymptotically approach a limiting value. For three-dimensional cases, Lyu [10] found that reducing the number of airfoil control points had a relatively more significant influence on the drag increase than decreasing the number of airfoil sections of the Common Research Model wing benchmark case. In contrast to the optimization with predefined variables, an adaptive scheme [11] was introduced that refines and coarsens the parameterization based on adjoint surface sensitivities in a multi-level gradient-based optimization process.

Global search approaches associated with high-accuracy simulations often cost numerous computational resources and make the searching iterations inefficient which could be even worse with increasing dimensionality. To investigate the impact of distribution of design variables and dimensionality on optimization designs, and to improve the performance of global search methods, design variables and adjustments of design space are determined beforehand based on adjoint surface sensitivity information which is usually calculated in gradient-based searching approaches for a profile-constrained optimization case. The proposed refinement strategy has the potential for guiding designers to reduce design dimensionality and get better solutions.

2 Model Problem

The optimization case [1] under study is the drag minimization of a modified NACA0012 in transonic inviscid flow with freestream Mach number (M) of 0.85 subject to a thickness (t) constraint at zero degrees incidence (α). The case is based on work done by Vassberg [12] with a slight modification to NACA0012 to make sure the trailing edge is sharp. The airfoil is defined as

$$y = \pm\frac{0.12}{0.2}\left(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\right)$$

Where, $x \in [0,1]$. The problem is formally described as

$$\begin{aligned} \text{Minimize: } & C_d \\ \text{Subject to: } & M = 0.85, \\ & \alpha = 0^\circ, \\ & t \geq t_{baseline} \quad \forall x \in [0,1] \end{aligned}$$

This benchmark case has been investigated substantially previously and a range of highly optimized drag results ranging from 32 to 86 drag counts were produced [2]. The only source of drag is that arises with any shocks and this case requires relatively moderate computational costs which make it appropriate to investigate the selection of design variables.

3 Shape Parameterization

Proper parameterization method applied to certain problem depends on the structure of the optimization framework, efficiency requirement of solutions and geometry characteristics that designers concern with. In addition, the determination of design variables is often subject to the parameterization method. Based on study of Samareh [13] and Masters [14], the Bèzier surface Free-form deformation (FFD) approach developed by Sederberg and Parry [15] proves to be one of well-performing parameterization schemes. It possesses several features: preservation of smoothness for surface grids; independence of geometry complexity; flexibility in determination of variables. A Bèzier surface FFD can be used in two-dimensional and three-dimensional space. For geometries in a plane, generally, a rectangular lattice of $(l+1)\times(m+1)$ uniformly spaced control points, $\mathbf{P}_{\mathrm{ij}}$, is placed around the initial geometry with a local coordinate system O'-STU. The control point positions are defined as

$$\mathbf{P}_{\mathrm{ij}} = \mathbf{O}' + \frac{i}{l}\mathbf{S} + \frac{j}{m}\mathbf{T}$$

For $i = 0,...,l$, $j = 0,...,m$. Thus the undeformed domain defined by the lattice is normalized to a unit domain by the notation of local coordinates. The displacement of any point in the lattice is associated with all control points which stays the same when a user-defined lattice is applied such to gain enhancement to the control of the geometry. A Newton iterative method is deployed in this paper to calculate local coordinates of deformed geometry within the customized lattice, as shown in Figure 1.

If the trivariate Bernstein polynomial function is used as base function in the FFD approach, new global coordinates of surface points with respect to the control point positions $\mathbf{P}_{\mathrm{ij}}$ is defined as

$$\mathbf{X} = \sum_{i=0}^{l}\binom{l}{i}(1-s)^{l-i}s^{i}\left(\sum_{j=0}^{m}\binom{m}{j}(1-t)^{m-j}t^{j}\mathbf{P}_{\mathrm{ij}}\right)$$

Preliminary drag resolution study shows that a 513×257 O-mesh (in Figure 1) captures drag to within 0.1 counts. The farfield resides 100 chord lengths away from the airfoil. Movement of control points is restricted to the *y*-direction and symmetry is preserved by pairing equivalent upper and lower control points symmetrically. There are 11 design variables (DV) initially, and two sets of fixed control points near the leading edge and trailing edge besides, as shown in Figure 1.

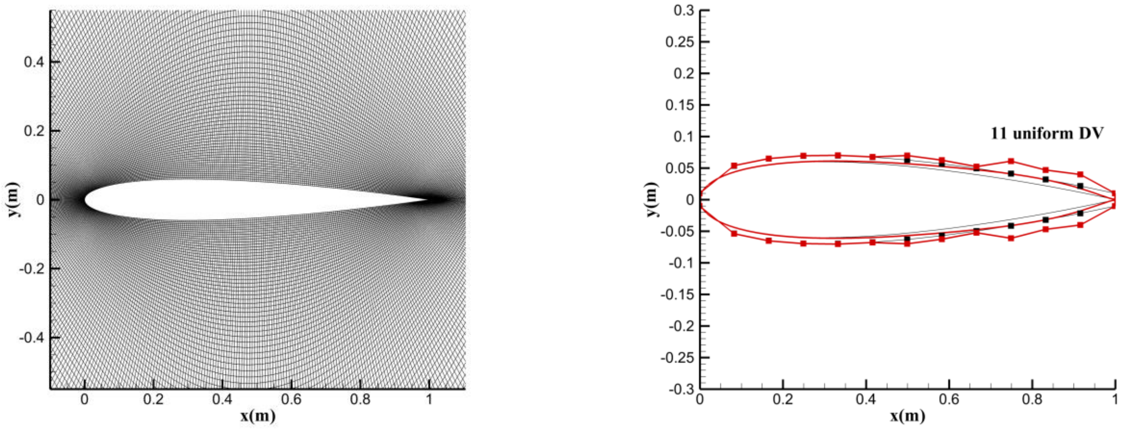


Figure 1: Close-up view of the 513×257 O-mesh (left) and stretched Bèzier surface control lattice in red line corresponding to 11 uniform design variables (right).

4 Optimization Framework

This inviscid compressible flow problem is investigated with SU2 software suite [16], a computational analysis and design package, which is capable to solve both the flow and adjoint systems of equations to provide high-quality flow simulation and sensitivity information that can be used for a gradient-based optimization design. SU2 flow solutions and calculation of surface sensitivity with discrete adjoint equation modules are performed for the case described above.

4.1 Aerodynamic Model

The numerical solver of the Euler equations implemented in SU2 suite is employed. The second-order JST scheme is used among several space discretization schemes provided by the software. An implicit Euler, local time-stepping and multigrid method of three levels are used to converge to the steady-state solution quickly, and the FGMRES method in conjunction with the LU-SGS preconditioner is used to solve the system.

4.2 Determination of Design Variables

Optimizers always pursue solutions with better aerodynamic characteristics while keeping less variables to make the design process efficient. Sensitivity data is usually calculated throughout gradient-based optimization process to obtain the local gradients which can dictate the surface deformation direction at a computational cost in the order of one flow solve. By solving the discrete adjoint equations [17] the sensitivity of drag coefficient with respect to the unit normal at each surface mesh point is obtained, i.e

$$\frac{\partial C_d}{\partial \boldsymbol{x}_i}$$

for unit normal $\boldsymbol{x}_i = \left[x_i, y_i \right]$.

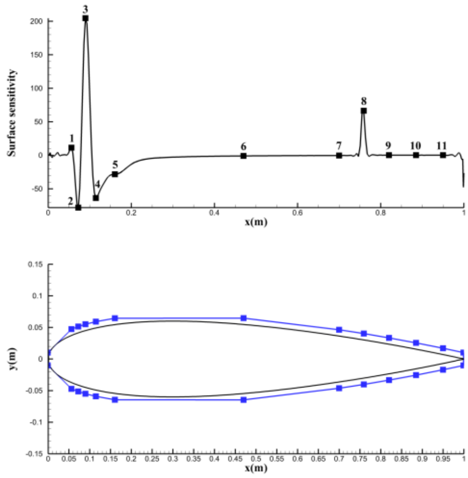


Figure 2: Customized control lattice of 11 design variables determined based on surface sensitivities.

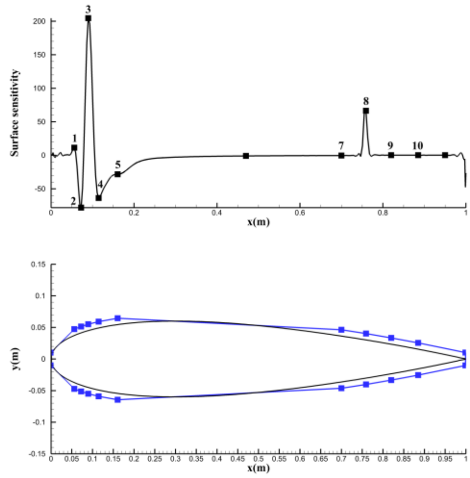


Figure 3: Customized control lattice of 9 design variables determined based on surface sensitivities.

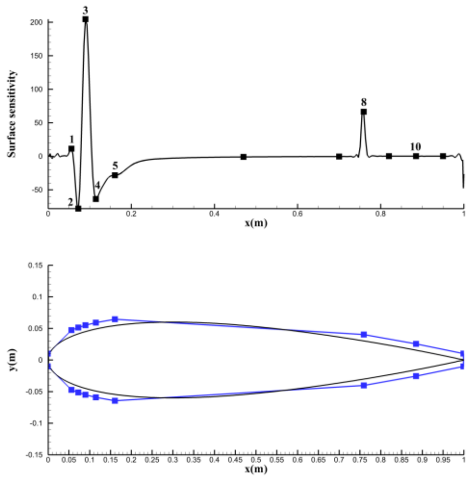


Figure 4: Customized control lattice of 7 design variables determined based on surface sensitivities.

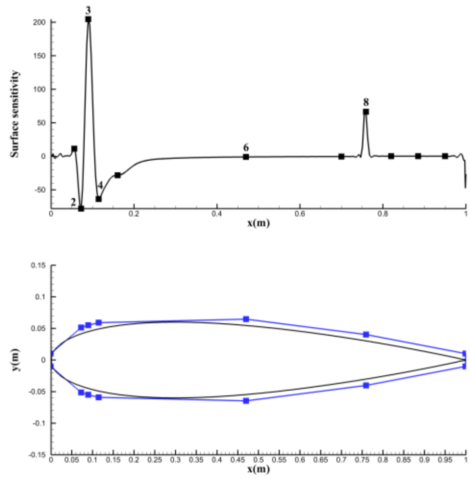


Figure 5: Customized control lattice of 5 design variables determined based on surface sensitivities.

Instead of calculating gradients further, surface sensitivities are analyzed and employed in global search framework to determine efficient design variables. Surface sensitivities along the wall are shown in Figure 2. For this transonic NACA0012 problem, inviscid flow field is dominated by a strong shock wave located near 75% chord length. Moreover, two high sensitivity areas located at 5% to 15% and 75% chord length are observed. Mesh points with high sensitivities may have significant impact on the flow characteristics when they are stretched through movements of control lattice. Therefore, control points are put around high sensitivity areas with dimensionality from 11 to 5 as depicted in Figures 2 to 5. Control points #7 and #9 are designed to control surface near the shock wave. Control point #11 brings geometry near the trailing edge under control, and point #6 together with #10 is expected to reinforce the control over the whole geometry.

For uniform distribution of design variables, search space is bounded within $\left[0, 0.02\right]^{DV}$ m. For nonuniform distribution of variables, search space is adjusted based on surface sensitivities to explore the promising space thoroughly. Specifically, control points near which surface sensitivities are negative are designed with larger space to raise the possibility of reaching global optimum, while for high sensitivity areas search space is contracted to construct more efficient surrogates and avoid inferior solutions. The adjusted design space is defined as

$$\mathbf{D} = 0.02 \cdot \left(1 - 0.2 \cdot rand \cdot \frac{\mathbf{S}}{S_{peak} - S_{valley}}\right)$$

Where $\mathbf{D} = \{D_1, D_2, ..., D_{DV}\}$ denotes design space and $\mathbf{S} = \{S_1, S_2, ..., S_{DV}\}$ is the surface sensitivity of the mesh point nearest to a certain control point. S_{peak} is the peak value of surface sensitivities, which is obtained at surface mesh point #3. S_{valley} denotes the valley value of sensitivities corresponding to that at mesh point #2.

5 Optimization Results

Optimization design starting from an initial NACA0012 has been performed in this work. The number of design variables gradually reduces from 11 to 5. The effects of nonuniform variables along with search space adjustments are demonstrated by comparison with optimization solutions of uniform design variables.

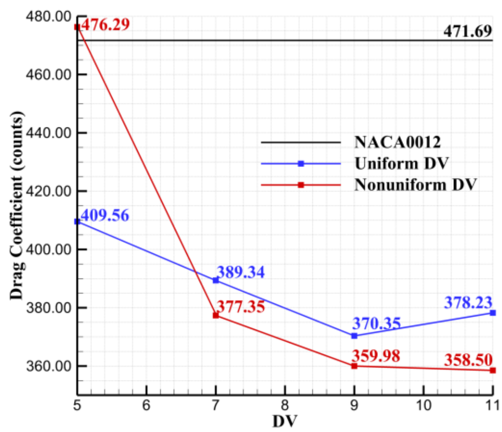


Figure 6: Results for the inviscid optimization of NACA0012 with increased number of variables.

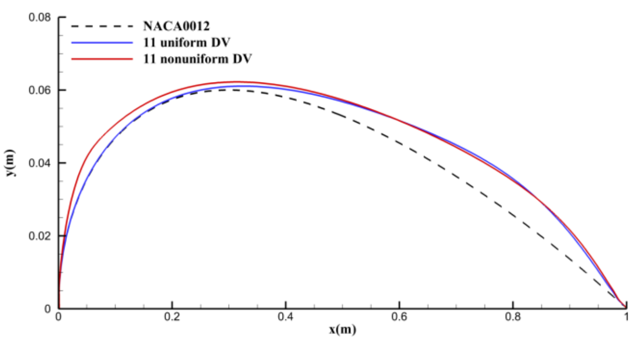
Figure 6 shows drag coefficients of optimum solutions with respect to number of design variables. It can be seen that, in general, the final results improve with increasing dimensionality. There is however one instance where this is not true. For uniform distribution of design variables, drag coefficient rises by 7.88 counts with dimensionality increasing from 9 to 11. This could be related to inferior training dataset of 11 uniform design variables, which worsens performance of Radial Basis

Function approximation models and besides, may impact on the performance of Multi-island GA module. It can also be seen that nonuniform distribution of design variables tends to result in solutions with lower drag, in particular the optimized shape with 11 nonuniform variables proved to be the best geometry in this work with drag coefficient of 358.50 counts, nearly 20 counts lower than that of uniform arrangement. When dimensionality reduces from 7 to 5, the advantage of refinement strategy of design variables expires, and the highest drag was obtained of 476.29 counts, which is even larger than the baseline drag coefficient of 471.69 counts. This shows that the adjustments of variables and design space may be not robust enough to handle low dimensionality under 7 for airfoil profiles. For the design with 5 variables, 3 control points are around the high sensitivity area near the nose, and only other two manipulate a majority part of the surface, which leads to strong linearity of surrogate construction and noneffective control lattice. Despite the failure with 5 variables, it is noteworthy that the case of 9 nonuniform variables has lower drag coefficient of 359.98 counts compared with that of 11 uniform design variables which is 378.23 counts. The refinement strategy could help make wiser decisions on choosing design variables to search optimal solutions more effectively and efficiently.

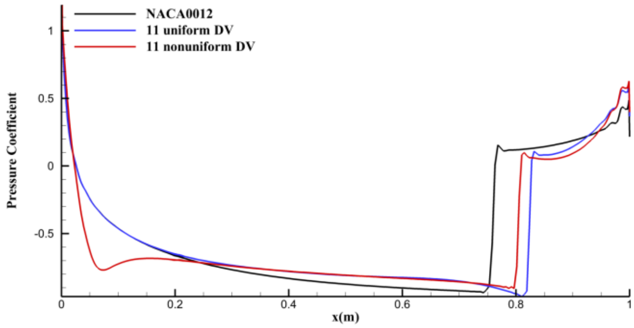
Optimal solutions are compared in Figures 7 to 10. With uniform design variables, optimal shapes are almost identical to NACA0012 at the forepart of about 0.2m. Then the discrepancy is getting larger and reaches extremum value of about 0.1m at 75% to 80% chord length. In general, optimized shapes with different uniform variables are similar to each other. As a result, the four solutions present similar pressure distributions wherein initial shock wave is put off. The locations of shock wave for optimal shapes are different. A strong shock wave that locates at about 0.83m for the case of 11 variables gradually moves forward as dimensionality declines.

For cases designed with nonuniform variables, optimum airfoil geometries and flow characteristics have significantly changed from uniform cases. Most optimal configurations are pushed outwards drastically from initial surface, especially at the forepart of about 40% chord length, which leads to a single suction peak at 0.07m. A pressure plateau is observed to extend to the shock wave area. Deformation near the initial shock wave is less significant compared to designs with uniform variables, and the optimized configurations keep moving towards initial NACA0012 with decreasing dimensionality. It can be seen from the pressure field that shock wave strength of the optimal shape with 5 variables is almost the same as the initial one and gradually weakens as dimensionality increases which results in the trend of drag coefficient in Figure 6.

Refinements of design variables associated with space adjustments applied in optimization design has improved aerodynamic characteristics of baseline geometry. In areas where surface sensitivity reaches extrema, the surface is stretched outwards further which makes the leading edge blunter and thus changes the pressure load. The strong shock wave imposed on NACA0012 is significantly weakened although the location is ahead of that in the design with uniform variables. It can be seen in Figure 10 that the three control points at the forepart didn’t deform the geometry effectively despite of their locations where surface sensitivities reach extrema. This phenomenon suggests that, to guarantee the effectivity of the determination strategy introduced in this work and the function of control lattice, dimensionality should be above 7 for optimization design of airfoil profiles.

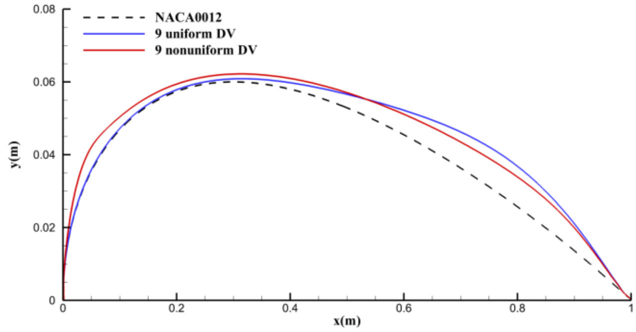


a) Optimum airfoil shapes

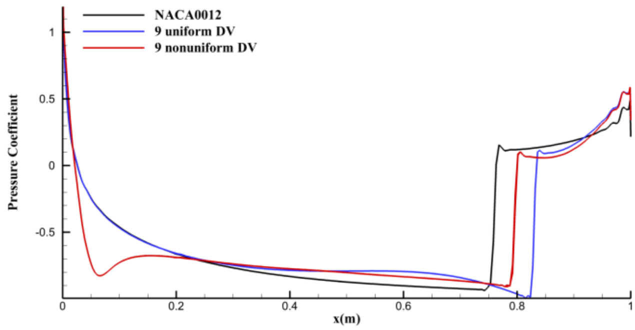


b) Pressure distributions for optimum airfoil shapes

Figure 7: Results of the optimization of a NACA0012 with 11 design variables.

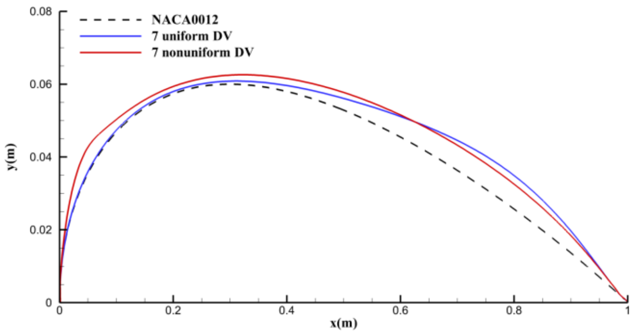


a) Optimum aerofoil shapes

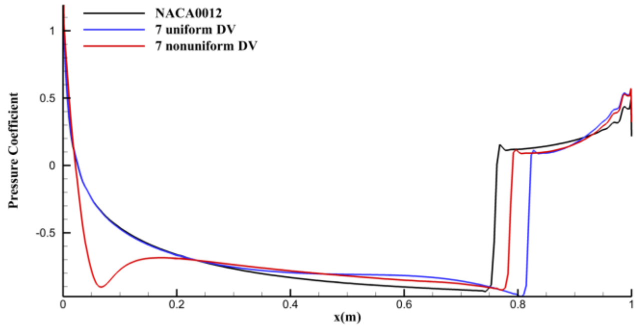


b) Pressure distributions for optimum airfoil shapes

Figure 8: Results of the optimization of a NACA0012 with 9 design variables.

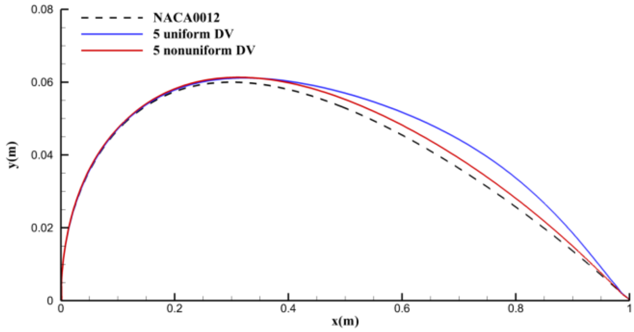


a) Optimum aerofoil shapes

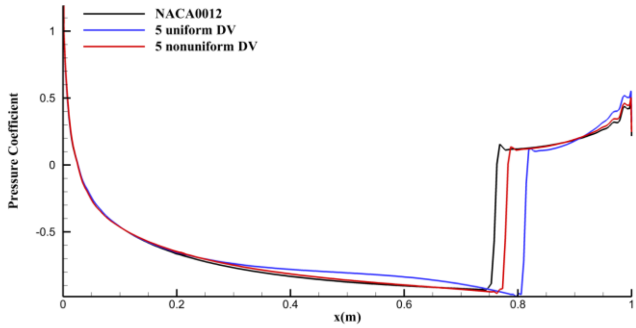


b) Pressure distributions for optimum airfoil shapes

Figure 9: Results of the optimization of a NACA0012 with 7 design variables.



a) Optimum aerofoil shapes



b) Pressure distributions for optimum airfoil shapes

Figure 10: Results of the optimization of a NACA0012 with 5 design variables.

6 Conclusion and Future Work

In this study, based on adjoint surface sensitivity information, a determination strategy of design variables and corresponding design space adjustment method are proposed and applied in global optimization design of NACA0012 airfoil. Specifically, FFD control points have been put near high sensitivity areas to investigate the impact of distribution of design variables and dimensionality on optimization designs. Effects of the proposed methods are verified by contrasting with designs from uniform distributed variables.

In general, drag reduces as dimensionality increases, except that for uniform distribution of design variables, drag coefficient rises by 7.88 counts with dimensionality increasing from 9 to 11, which is caused by inferior training dataset.

The refinement strategy introduced in this work results in solutions with lower drag compared with cases of uniform design variables under the same dimensionality and helps get improved aerodynamic characteristics using fewer variables. Moreover, it may possess capacity to resist the impact of inferior observational data. The best geometry in this work is obtained with 11 nonuniform design variables of which the drag coefficient is 358.50 counts, 19.73 counts lower than that of uniform arrangement. Optimal shapes with nonuniform design variables are pushed outwards drastically, especially at the forepart of about 40% chord length, which makes the leading edge blunter and changes the pressure field. In the area near initial shock wave, the optimal configurations keep moving towards baseline geometry when dimensionality reduces, while the optimal shapes with uniform variables are much more stable. This indicates that there is a close correlation between the effectivity of the strategy and dimensionality.

In the future, the refinement strategy of design variables based on adjoint sensitivities will be further investigated with increased dimensionality. Improving the adjustments of design space is expected so that global optimum would be reached more efficiently.

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