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Comparisons between direct simulation and penalizations methods for flow in a porous-fluid system

Charles-Henri Bruneau[1], Didier Lasseux[2], Francisco Valdés-Parada[3]
Corresponding author: bruneau@math.u-bordeaux.fr

[1] Univ. Bordeaux IMB, INRIA Bordeaux Sud-Ouest MEMPHIS, CNRS UMR5251, Talence 33405, France
[2] Univ. Bordeaux I2M, CNRS UMR5295, Talence 33405, France
[3] UAM , Mexico D.F. 09340, Mexico

Abstract: This work is devoted to a numerical study of two-dimensional incompressible flow around a porous medium obstacle on top of an impermeable wall. The porous medium is either formed of thousands of small particles for the direct numerical simulation or considered as a continuum and is modelled with penalized Navier-Stokes equations. Several penalization models are studied and simulated. Applications encompass flow at various regimes over a large porous rectangle. Results show significant discrepancies between the different penalization models compared to the direct numerical simulation of the flow inside and outside the porous medium.

Keywords: Porous media, Direct numerical simulation, Penalization models.

1 Introduction

For more than fifty years now, numerous analyses have been dedicated to the modelling of a porous medium-fluid system, either deriving a boundary condition at the interface [1] or using a simplified model within which the porous medium is taken into account using a penalization method [2, 3] in order to avoid numerical simulation of coupled equations in the fluid and in the porous medium without knowing the appropriate conditions at the interface.

In this work, the porous medium is a large rectangle made of many particles close to each others and the aim is to compute accurately incompressible flow inside and outside the porous medium. The flow is first computed by solving the Navier-Stokes equations in the whole domain including the particles. In a second step, the porous zone is replaced by an homogeneous medium taking into account its property ("permeability" and porosity). Several models are proposed adding a penalization term inside the momentum equation and results are compared to those obtained by direct numerical simulation. The penalization models range from the simple first order model to a full model involving a tensor depending on the local flow. They can be found respectively in [2, 4, 3, 5, 6].

The Navier-Stokes equations are approximated by an accurate finite differences scheme and solved by a multigrid procedure involving several grid levels. The code is highly parallelized with MPI directives.

The models and the results are carefully analysed to see which penalized model yields results closest to those from the direct simulation. In particular the flow inside the porous medium are scrutinised and the velocity profiles are provided in the whole domain to highlight the impact of penalization approach close to and far from the porous zone.

2 Numerical approximation and results

The porous rectangle is made of 12000 solid squared particles aligned on 30 rows of 400 squares with a porosity $\epsilon = 0.913$ that is consistent with Brinkman's approximation. The domain is a large box $\Omega = (0, 600) \times (0, 600)$ including the porous rectangle $(100, 500) \times (0, 30)$. To better compare the results, the numerical simulations are performed on a medium 3840×3840 cells Cartesian grid. The genuine Navier-Stokes equations are solved for the velocity U and pressure p in the fluid:

$$\begin{aligned} \rho\partial_t U + \rho(U\cdot\nabla)U - \mu\Delta U + \nabla p &= 0 \quad in\ \Omega_T = \Omega\times(0,T), \\ div\, U &= 0 \quad in\ \Omega_T, \end{aligned}$$

where ρ is the density, μ is the viscosity and T is the simulation time. When a penalized model is used to account for the porous medium, the momentum equation is replaced by:

$$\rho\partial_t U + \rho(U\cdot\nabla)U - \mu\Delta U + \epsilon\mu H(U)^{-1}U + \nabla p \;= 0 \quad in\ \Omega_T = \Omega\times(0,T),$$

where $H(U)$ is a tensor which can depend on U [5].
The simplest model is obtained while considering $H(U) = kI$ where k is the intrinsic permeability of the porous medium and I the identity tensor (this case is noted K in the following). Then the well-known second order extension of [3] with

$$\epsilon\mu H(U)^{-1}U = \frac{\epsilon\mu}{k}U + \frac{\rho\epsilon^2 F}{sqrt(k)}\|U\|U$$

where F is the Forchheimer tensor (this case is denoted K2). Finally the more complex form of $H(U)$, depending on the local Reynolds number and on the angle of the pressure gradient inside the porous medium, is used to improve the approximation [6]. This last case is referred as H and is compared to the two simpler cases K and K2.

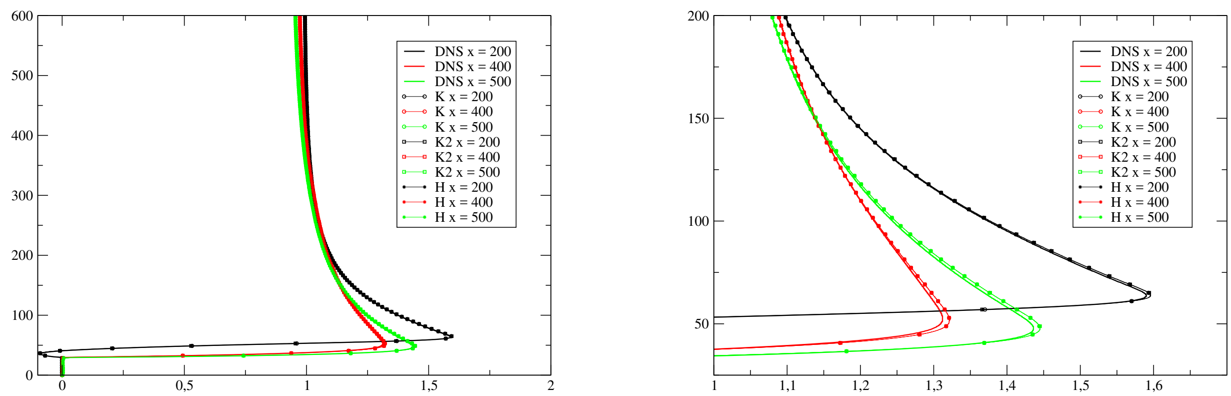

Figure 1: Vertical mean velocity profile at three different positions in x-direction 200, 400 and 500 for $Re = 10$. Left: the direct numerical simulation (DNS) is compared to the penalization models (K), (K2) and (H). Right: zoom of the velocity profile above the porous zone.

At low Reynolds number, $Re = 10$, the four approximations above give about the same profile in the fluid domain as can be seen in Figure 1. The velocity obtained by the three penalization models almost coincide between each other and are in very good agreement with the mean velocity profile obtained from the direct numerical simulation. There is only a small discrepancy at the velocity maximum. Let us point out that for this low Reynolds number the three penalization models give almost the same dispersion tensor as the correction is negligible in front of the main term.
For higher Reynolds numbers there are more significant discrepancies. Indeed at $Re = 100$ the three penalization models do not give any more the same result. Nevertheless there are in quite good agreement with the reference flow computed by DNS on a finer grid (see Figure 2). But the results obtained with the full tensor (H) are closer to the reference flow which shows the efficiency of the correction. In that case the interest of the penalization model (H) is obvious as it gives almost the same result than DNS on a finer grid. Thus this is a way to compute the right solution saving a lot of computational time.

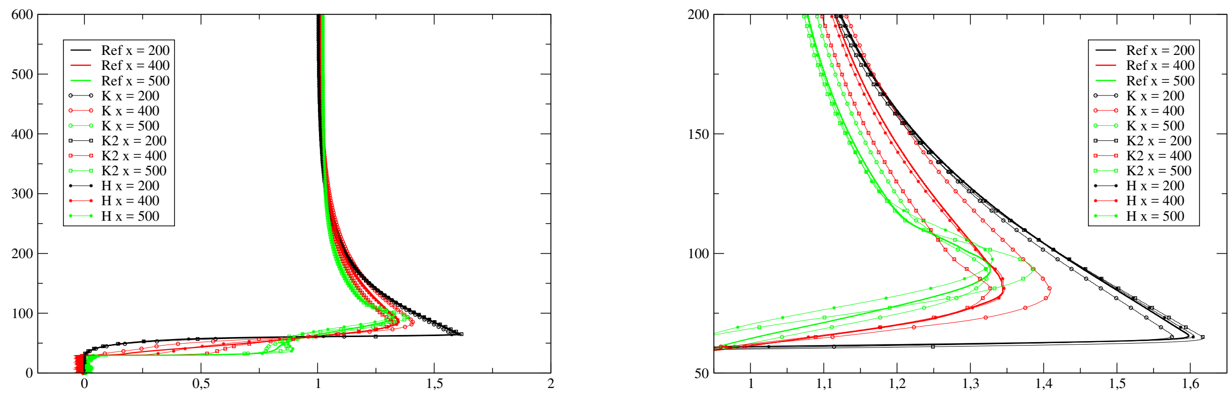

Figure 2: Vertical mean velocity profile at three different positions in x-direction 200, 400 and 500 for $Re = 100$. Left: the direct numerical simulation (Ref) is compared to the penalization models (K), (K2) and (H). Right: zoom of the velocity profile above the porous zone. In this case the DNS is performed on a finer grid ans is referred as the reference flow.

3 Conclusions

Results obtained in this work show that an incompressible flow within a porous medium-fluid system can be successfully modelled by a volume penalization term using a tensor depending on both the local Reynolds number and the local pressure gradient orientation.

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